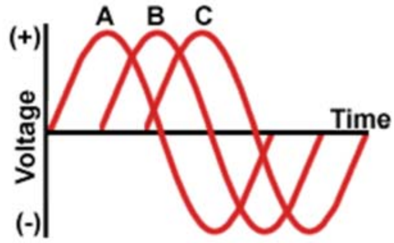
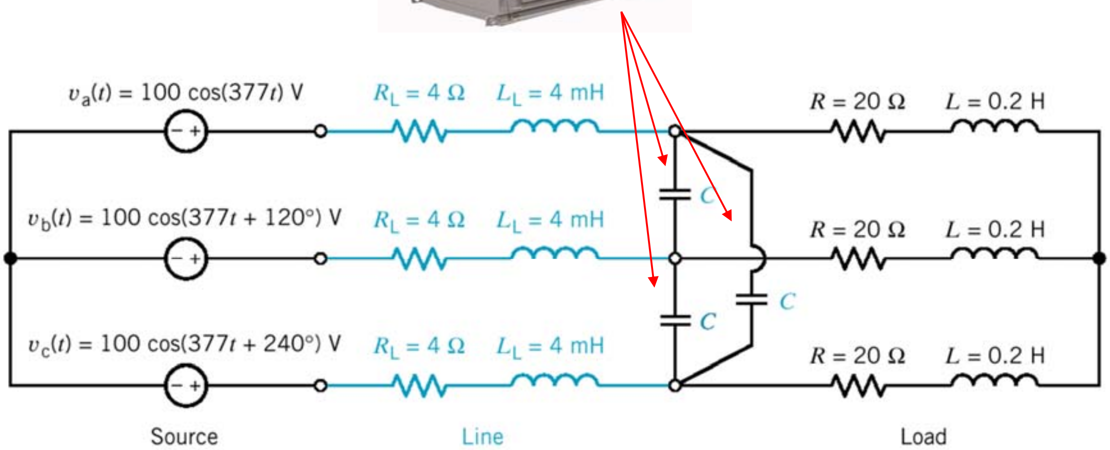
Chapter 12

Three-Phase Circuits



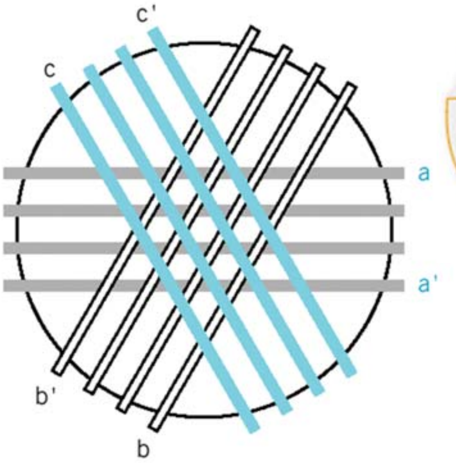
Power Factor Correction

Medium Voltage Metal Enclosed Capacitor Banks

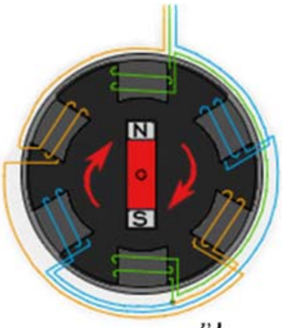


A balanced three-phase circuit

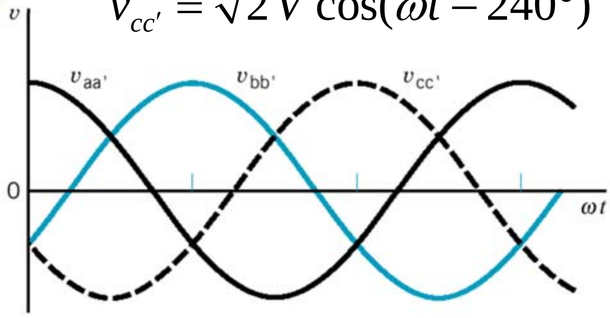
Three-Phase Voltages



(a)



$$v_{aa'} = \sqrt{2}V\cos\omega t$$
$$v_{bb'} = \sqrt{2}V\cos(\omega t - 120°)$$
$$v_{cc'} = \sqrt{2}V\cos(\omega t - 240°)$$

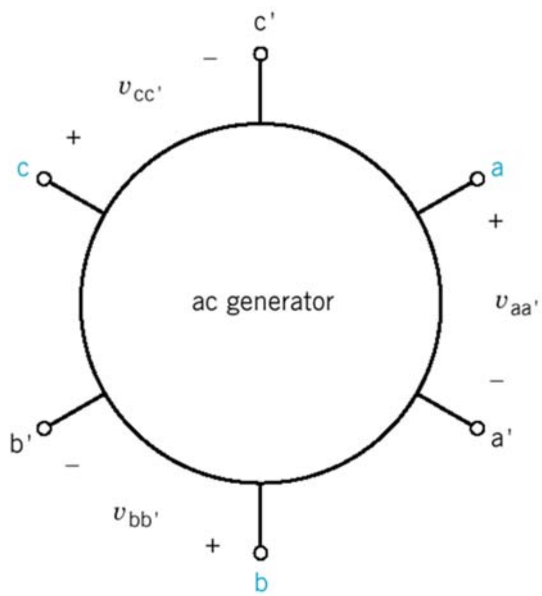


(b)

(a) The three windings on a cylindrical drum used to obtain three-phase voltages

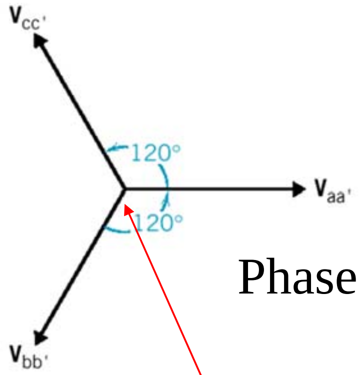
(*b*) Balanced three-phase voltages

Three-Phase Voltages



Generator with six terminals

Three-Phase Balanced Voltages

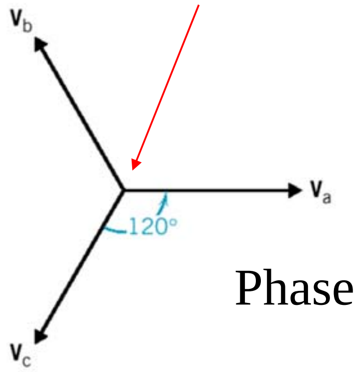


$$\mathbf{V}_{aa'} = V\angle 0°$$
$$\mathbf{V}_{bb'} = V\angle -120°$$
$$\mathbf{V}_{cc'} = V\angle -240° = V\angle +120°$$

Phase sequence or phase rotation is ***abc***

Positive Phase Sequence

neutral terminal

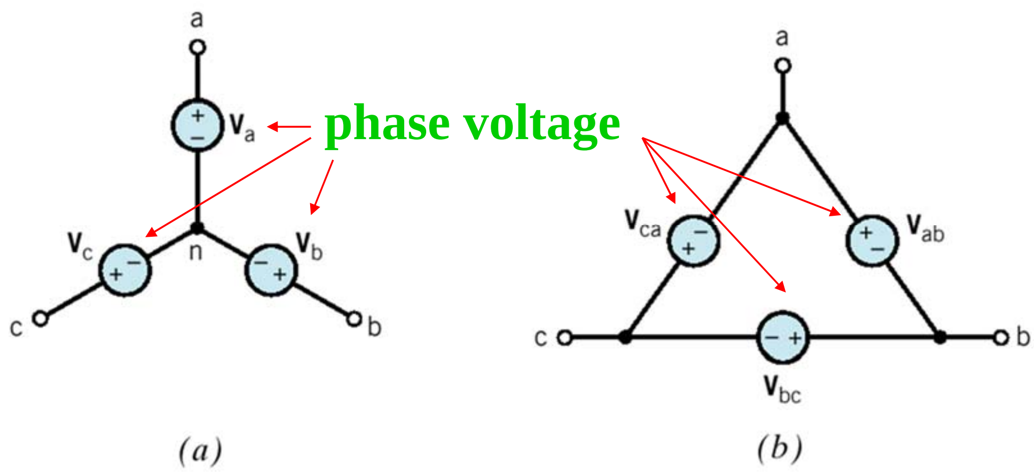


$$\mathbf{V}_{a} = V\angle 0°$$
$$\mathbf{V}_{c} = V\angle -120°$$
$$\mathbf{V}_{b} = V\angle -240° = V\angle +120°$$

Phase sequence or phase rotation is ***acb***

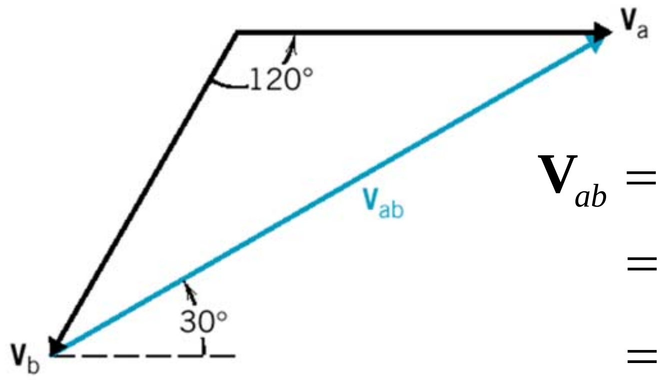
Negative Phase Sequence

Two Common Methods of Connection



(*a*) Y-connected sources (b) Δ-connected sources

Phase and Line Voltages



The line-to-line voltage $\mathbf{V}_{ab}$ of the Y-connected source

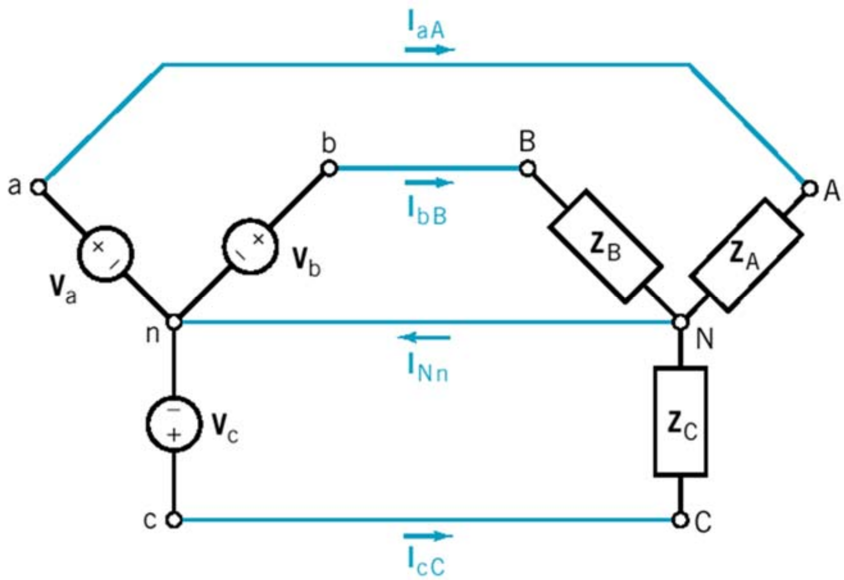
$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_a - \mathbf{V}_b \\ &= V_p\angle 0° - V_p\angle -120° \\ &= V_p - V_p(-0.5 - j0.866) \\ &= \sqrt{3}V_p\angle 30°\end{aligned}$$

Similarly

$$\mathbf{V}_{bc} = \sqrt{3}V_p\angle -90°$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -210°$$

The Y-to-Y Circuit



A four-wire Y-to-Y circuit

The Y-to-Y Circuit (cont.)

Four - wire $\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A}, \mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B}, \text{ and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C}$

$$\mathbf{I}_{nN} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC}$$

The average power delivered by the three-phase source to the three-phase load

$$P = P_A + P_B + P_C$$

When $Z_A = Z_B = Z_C$ the load is said to be ***balanced***

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{V\angle 0°}{Z\angle\theta}, \mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B} = \frac{V\angle -120°}{Z\angle\theta}, \text{ and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C} = \frac{V\angle 120°}{Z\angle\theta}$$

$$\mathbf{I}_{aA} = \frac{V}{Z}\angle -\theta°, \mathbf{I}_{bB} = \frac{V}{Z}\angle(-\theta - 120°), \text{ and } \mathbf{I}_{cC} = \frac{V}{Z}\angle(-\theta + 120°)$$

The Y-to-Y Circuit(cont.)

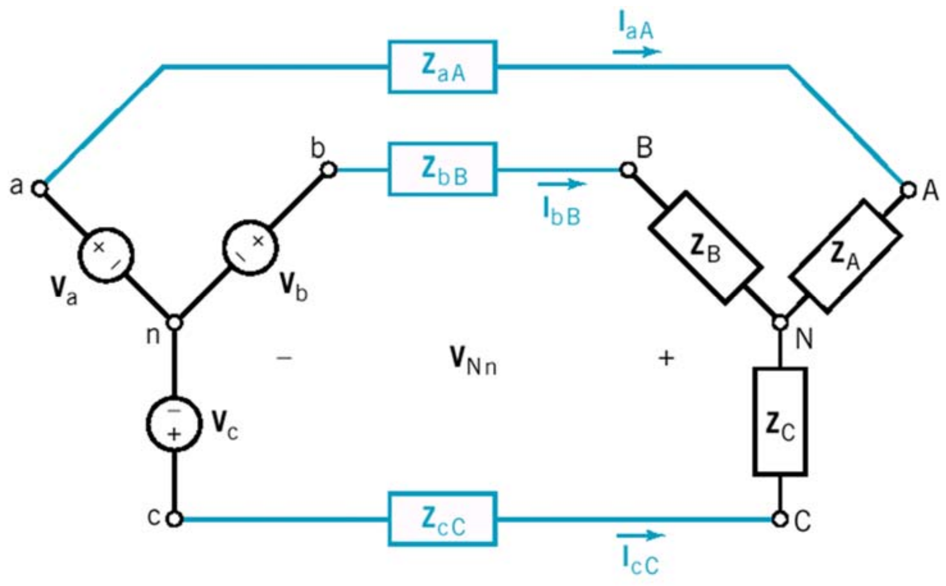
There is no current in the wire connecting the neutral node of the source to the neutral node of the load.

$$\mathbf{I}_{nN} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

The average power delivered to the load is

$$\begin{aligned} P &= P_A + P_B + P_C \\ &= V\frac{V}{Z}\cos(-\theta) + V\frac{V}{Z}\cos(-\theta) + V\frac{V}{Z}\cos(-\theta) \\ &= 3\frac{V^2}{Z}\cos(\theta) \end{aligned}$$

The Y-to-Y Circuit (cont.)



A three-wire Y-to-Y circuit

The Y-to-Y Circuit (cont.)

Three - wire

We need to solve for V_{Nn}

$$0 = \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A} + \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B} + \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

$$= \frac{V\angle 0° - \mathbf{V}_{Nn}}{\mathbf{Z}_A} + \frac{V\angle -120° - \mathbf{V}_{Nn}}{\mathbf{Z}_B} + \frac{V\angle 120° - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

Solve for V_{Nn}

$$\mathbf{V}_{Nn} = \frac{(V\angle -120°)\mathbf{Z}_A\mathbf{Z}_C + V\angle 120°\mathbf{Z}_A\mathbf{Z}_B + V\angle 0°\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A},\ \mathbf{I}_{bB} = \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B},\ \text{and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

The Y-to-Y Circuit (cont.)

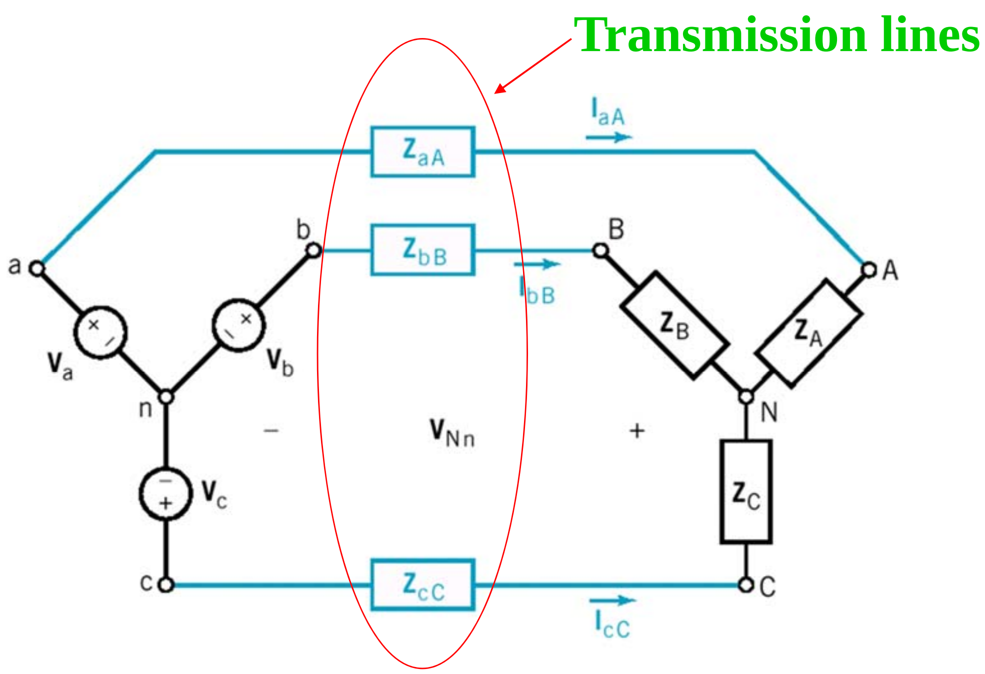
When the circuit is ***balanced*** ***i.e.*** $Z_A = Z_B = Z_C$

$$\mathbf{V}_{Nn} = \frac{(V\angle -120°)\mathbf{ZZ} + V\angle 120°\mathbf{ZZ} + V\angle 0°\mathbf{ZZ}}{\mathbf{ZZ}+\mathbf{ZZ}+\mathbf{ZZ}}$$
$$= 0$$

The average power delivered to the load is

$$P = P_A + P_B + P_C$$
$$= 3\frac{V^2}{Z}\cos(\theta)$$

The Y-to-Y Circuit (cont.)

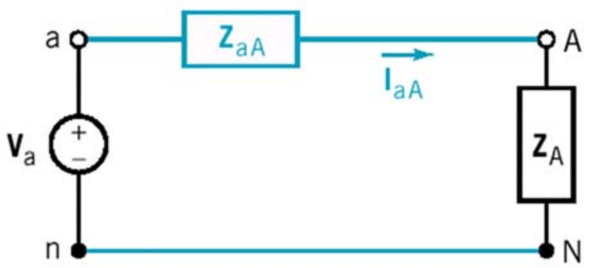


A three-wire Y-to-Y circuit with line impedances

The Y-to-Y Circuit (cont.)

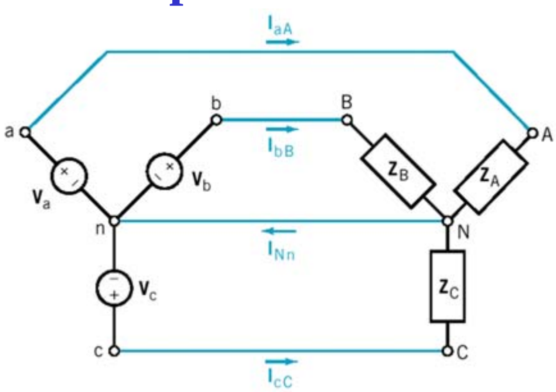
The analysis of ***balanced Y-Y circuits*** is ***simpler*** than the analysis of ***unbalanced Y-Y circuits***.

- V_{Nn} = 0. It is not necessary to solve for V_{Nn}.
- The line currents have equal magnitudes and differ in phase by 120 degree.
- Equal power is absorbed by each impedance.



Per-phase equivalent circuit

Example 12.4-1 S = ?



Unbalanced 4-wire

$$\mathbf{V}_a = 110\angle 0^\circ \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_b = 110\angle -120^\circ \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_c = 110\angle 120^\circ \quad \text{V}_{\text{rms}}$$

$$\mathbf{Z}_A = 50 + j80 \quad \Omega$$
$$\mathbf{Z}_B = j50 \quad \Omega$$
$$\mathbf{Z}_C = 100 + j25 \quad \Omega$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{110\angle 0^\circ}{50 + j80} = 1.16\angle -58^\circ \quad \text{A}_{\text{rms}}$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B} = \frac{110\angle -120^\circ}{j50} = 2.2\angle 150^\circ \quad \text{A}_{\text{rms}}$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C} = \frac{110\angle 120^\circ}{100 + j25} = 1.07\angle -106^\circ \quad \text{A}_{\text{rms}}$$

Example 12.4-1 (cont.)

$$\mathbf{S}_A = \mathbf{I}_{aA}^* \mathbf{V}_a = 68 + j109 \quad \text{VA}$$
$$\mathbf{S}_B = \mathbf{I}_{bB}^* \mathbf{V}_b = j242 \quad \text{VA}$$
$$\mathbf{S}_C = \mathbf{I}_{cC}^* \mathbf{V}_c = 114 + j28 \quad \text{VA}$$

The total complex power delivered to the three-phase load is

$$\mathbf{S} = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 182 + j379 \quad \text{VA}$$

Example 12.4-2 S = ? Balanced 4-wire

$$\mathbf{V}_a = 110\angle 0° \quad \text{V}_{\text{rms}} \qquad \mathbf{Z}_A = 50 + j80 \quad \Omega$$
$$\mathbf{V}_b = 110\angle -120° \quad \text{V}_{\text{rms}} \qquad \mathbf{Z}_B = 50 + j80 \quad \Omega$$
$$\mathbf{V}_c = 110\angle 120° \quad \text{V}_{\text{rms}} \qquad \mathbf{Z}_C = 50 + j80 \quad \Omega$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{110\angle 0°}{50 + j80} = 1.16\angle -58° \quad \text{A}_{\text{rms}}$$

$$\mathbf{S}_A = \mathbf{I}_{aA}^* \mathbf{V}_a = 68 + j109 \quad \text{VA}$$

The total complex power delivered to the three-phase load is

$$\mathbf{S} = 3\mathbf{S}_A = 204 + j326 \quad \text{VA}$$

Also

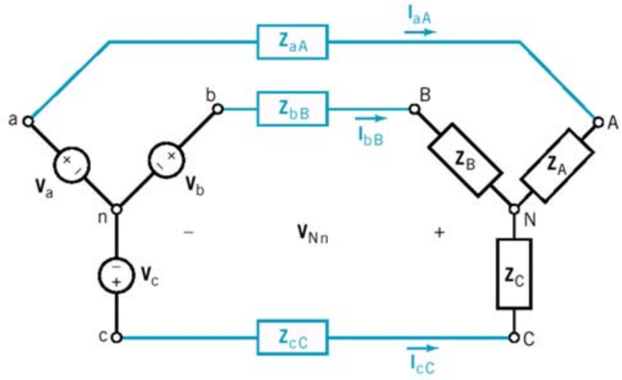
$$\mathbf{I}_{bB} = 1.16\angle -177° \quad \text{A}_{\text{rms}}, \mathbf{I}_{cC} = 1.16\angle 62° \quad \text{A}_{\text{rms}}$$

$$\mathbf{S}_B = 68 + j109 \quad \text{VA} = \mathbf{S}_C$$

Example 12.4-3 S = ?

$$\mathbf{V}_a = 110\angle 0° \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_b = 110\angle -120° \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_c = 110\angle 120° \quad \text{V}_{\text{rms}}$$

$$\mathbf{Z}_A = 50 + j80 \quad \Omega$$
$$\mathbf{Z}_B = j50 \quad \Omega$$
$$\mathbf{Z}_C = 100 + j25 \quad \Omega$$



Unbalanced 3-wire

Determine V_{Nn}

$$\mathbf{V}_{Nn} = \frac{(110\angle -120°)\mathbf{Z}_A\mathbf{Z}_C + 110\angle 120°\mathbf{Z}_A\mathbf{Z}_B + 110\angle 0°\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C}$$

$$= 56\angle -151° \quad \text{V}_{\text{rms}}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A}, \; \mathbf{I}_{bB} = \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B}, \text{ and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

Example 12.4-3 (cont.)

$$\mathbf{I}_{aA} = 1.71\angle -48°, \; \mathbf{I}_{bB} = 2.45\angle 3°, \text{ and } \mathbf{I}_{cC} = 1.19\angle 79°$$

$$\mathbf{S}_A = \mathbf{I}_{aA}^*\mathbf{V}_a = \mathbf{I}_{aA}^*(\mathbf{I}_{aA}\mathbf{Z}_A) = 146 + j234 \quad \text{VA}$$
$$\mathbf{S}_B = \mathbf{I}_{bB}^*\mathbf{V}_b = \mathbf{I}_{bB}^*(\mathbf{I}_{bB}\mathbf{Z}_B) = j94 \quad \text{VA}$$
$$\mathbf{S}_C = \mathbf{I}_{cC}^*\mathbf{V}_c = \mathbf{I}_{cC}^*(\mathbf{I}_{cC}\mathbf{Z}_C) = 141 + j35 \quad \text{VA}$$

The total complex power delivered to the three-phase load is

$$\mathbf{S} = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 287 + j364 \quad \text{VA}$$

Example 12.4-4 S = ? Balanced 3-wire

$$\mathbf{V}_a = 110\angle 0° \quad V_{rms} \qquad \mathbf{Z}_A = 50 + j80 \quad \Omega$$
$$\mathbf{V}_b = 110\angle -120° \quad V_{rms} \qquad \mathbf{Z}_B = 50 + j80 \quad \Omega$$
$$\mathbf{V}_c = 110\angle 120° \quad V_{rms} \qquad \mathbf{Z}_C = 50 + j80 \quad \Omega$$

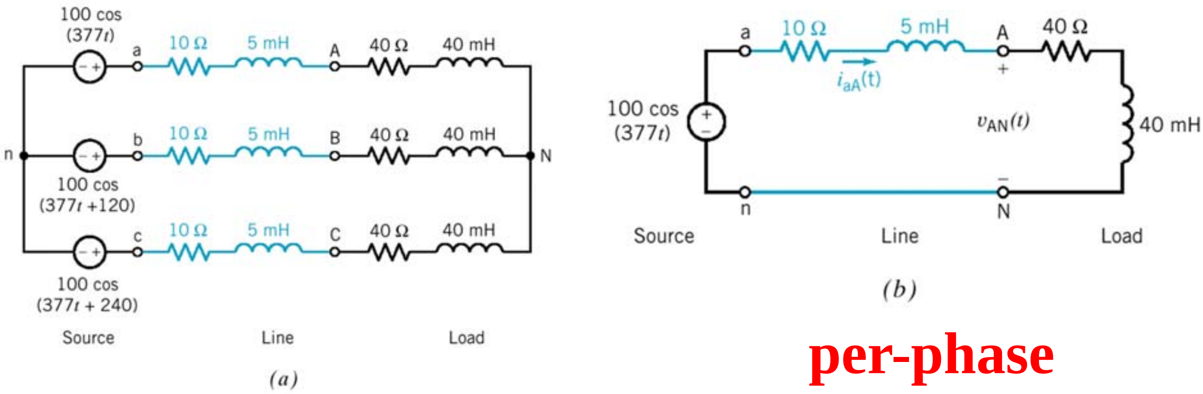
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{110\angle 0°}{50 + j80} = 1.16\angle -58° \quad A_{rms}$$

$$\mathbf{S}_A = \mathbf{I}_{aA}^{*}\mathbf{V}_a = 68 + j109 \quad VA$$

The total complex power delivered to the three-phase load is

$$\mathbf{S} = 3\mathbf{S}_A = 204 + j326 \quad VA$$

Example 12.4-5 P_{Load} = ? P_{Line} = ? P_{Source} = ?



per-phase equivalent circuit

Balanced 3-wire

$$\mathbf{I}_{aA}(\omega) = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{100\angle 0°}{50 + j(377)(0.045)} = 1.894\angle -18.7° \quad A$$

The phase voltage at the load is

$$\mathbf{V}_{AN}(\omega) = (40 + j(377)(0.04))\mathbf{I}_{aA}(\omega) = 81\angle 2° \quad V$$

Example 12.4-5 (cont.)

The power delivered by the source is

$$P_a = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I)$$

$$= \frac{(100)(1.894)}{2}\cos(18.7°) = 89.7 \quad \text{W}$$

The power delivered to the load is

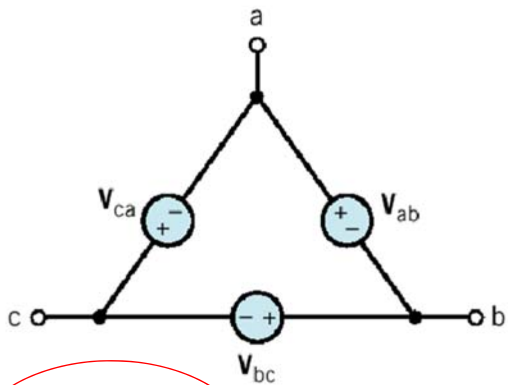
$$P_A = \frac{I_m^2}{2}\text{Re}(\mathbf{Z}_A) = \frac{(1.894)^2}{2}40 = 71.7 \quad \text{W}$$

The power lost in the line is

$$P_{aA} = \frac{I_m^2}{2}\text{Re}(\mathbf{Z}_{Line}) = \frac{(1.894)^2}{2}10 = 17.9 \quad \text{W}$$

Line loss ≈ 20%

The Δ-Connected Source and Load



$$\mathbf{V}_{ab} = 120\angle 0° \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_{bc} = 120.1\angle -121° \quad \text{V}_{\text{rms}}$$
$$\mathbf{V}_{ca} = 120.2\angle 121° \quad \text{V}_{\text{rms}}$$

Circulating current →

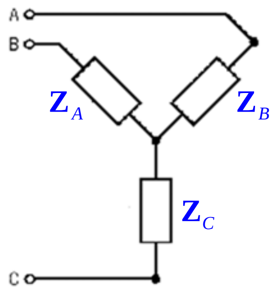
$$\mathbf{I} = \frac{(\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca})}{1} \approx -3.75 \quad \text{A}$$

Unacceptable

Total resistance around the loop

Therefore the Δ sources connection is seldom used in practice.

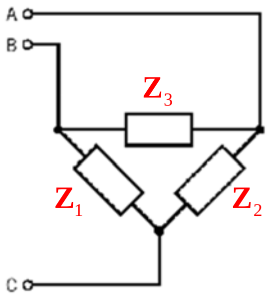
The Δ-Y and Y- Δ Transformation



$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

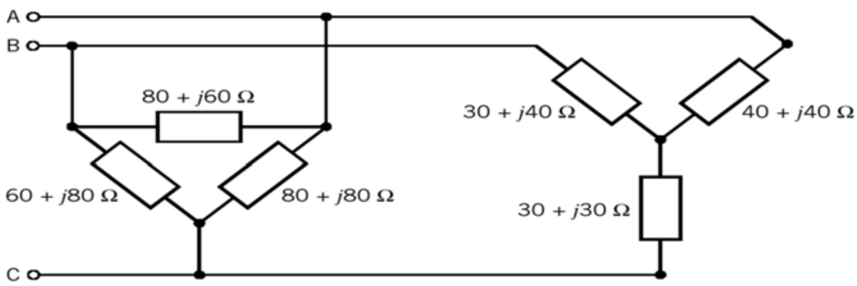


$$\mathbf{Z}_1 = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_B}$$

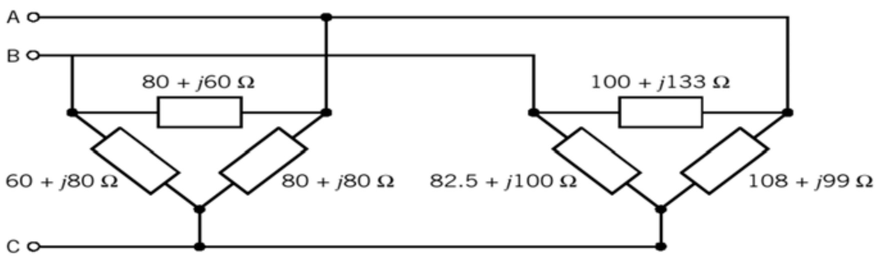
$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_C}$$

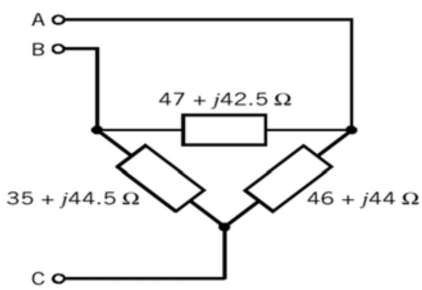
Example



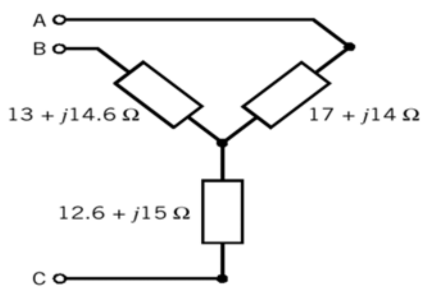
(a)



(b)

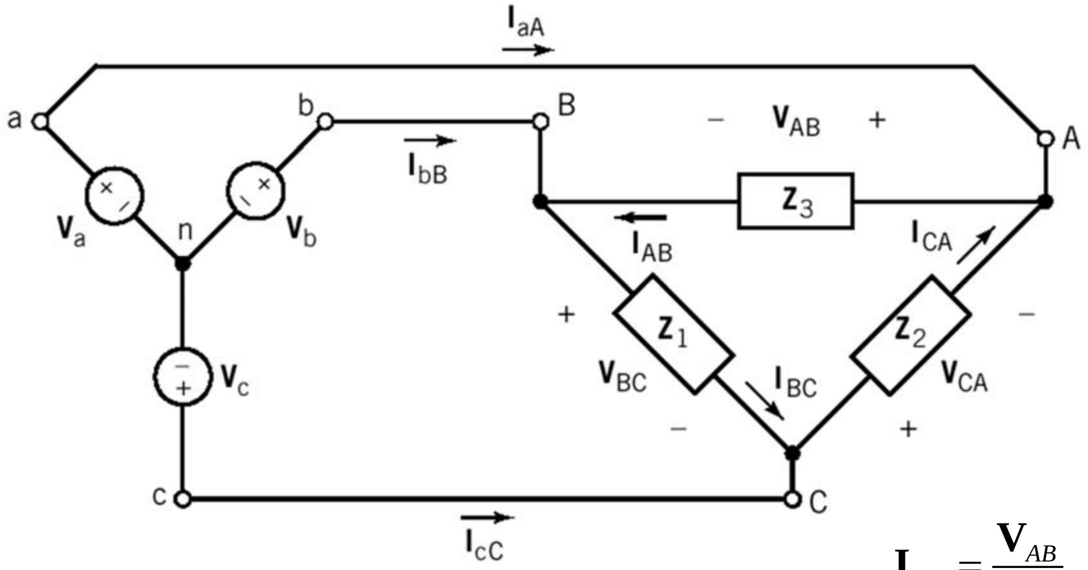


(c)



(d)

The Y- Δ Circuits

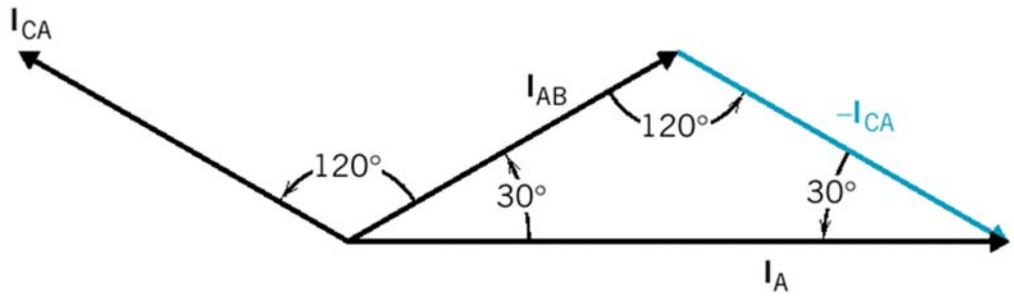


$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$
$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$$
$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

where

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_3}$$
$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_1}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_2}$$

The Y- Δ Circuits (cont.)

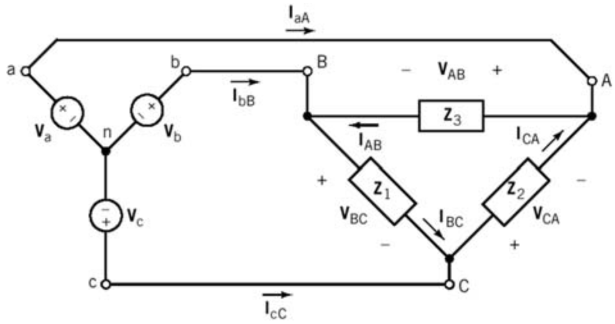


$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$
$$= I\cos\phi + j\sin\phi - I\cos(\phi + 120°) - j\sin(\phi + 120°)$$
$$= \sqrt{3} I \angle(\phi - 30°)$$

or

$$|\mathbf{I}_{aA}| = \sqrt{3}|I| \quad \Rightarrow \quad I_L = \sqrt{3} I_p$$

Example 12.6-1 $I_P = ?$ $I_L = ?$



$$\mathbf{V}_a = \frac{220}{\sqrt{3}}\angle -30^\circ \quad \mathrm{V_{rms}}$$

$$\mathbf{V}_b = \frac{220}{\sqrt{3}}\angle -150^\circ \quad \mathrm{V_{rms}}$$

$$\mathbf{V}_c = \frac{220}{\sqrt{3}}\angle 90^\circ \quad \mathrm{V_{rms}}$$

The **Δ**-connected load is balanced with $\mathbf{Z}_\Delta = 10\angle 50^\circ$

$$\mathbf{V}_{AB} = \mathbf{V}_a - \mathbf{V}_b = 220\angle 0^\circ \quad \mathrm{V_{rms}}$$

$$\mathbf{V}_{BC} = \mathbf{V}_b - \mathbf{V}_c = 220\angle -120^\circ \quad \mathrm{V_{rms}}$$

$$\mathbf{V}_{CA} = \mathbf{V}_c - \mathbf{V}_a = 220\angle -240^\circ \quad \mathrm{V_{rms}}$$

$$\Rightarrow$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = 22\angle 50^\circ \quad \mathrm{A_{rms}}$$

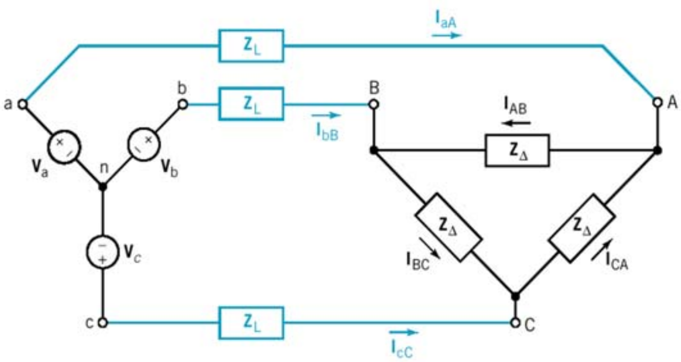
$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = 22\angle -70^\circ \quad \mathrm{A_{rms}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = 22\angle -190^\circ \quad \mathrm{A_{rms}}$$

The line currents are

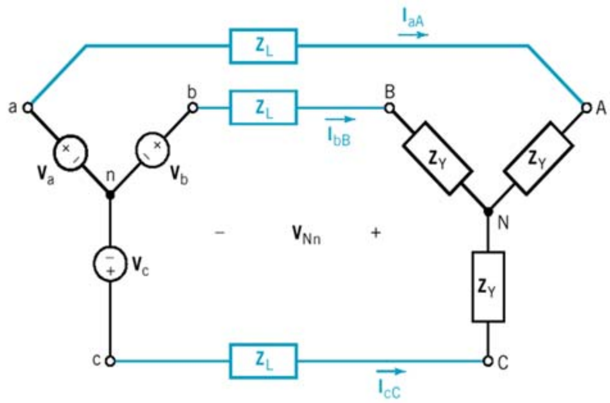
$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 22\sqrt{3}\angle 20^\circ,\ \mathbf{I}_{bB} = 22\sqrt{3}\angle -100^\circ,\ \mathbf{I}_{cC} = 22\sqrt{3}\angle -220^\circ$$

The Balanced Three-Phase Circuits



(a)

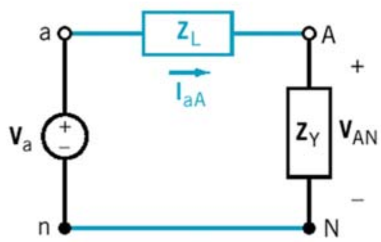
Y-to-Δ circuit



(b)

equivalent Y-to-Y circuit

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

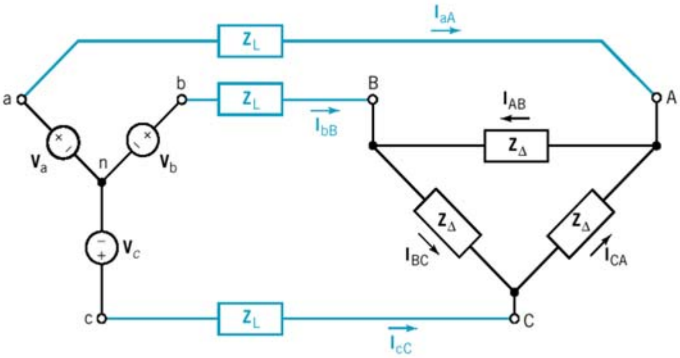


(c)

per-phase equivalent circuit

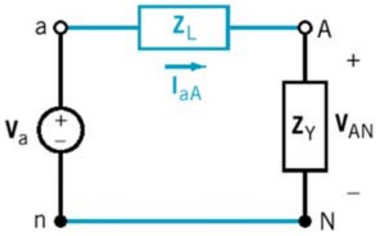
Example 12.7-1 $I_P = ?$

$$\mathbf{V}_a = 110\angle 0^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_b = 110\angle -120^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_c = 110\angle 120^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{Z}_L = 10 + j5 \quad \Omega$$
$$\mathbf{Z}_\Delta = 75 + j225 \quad \Omega$$

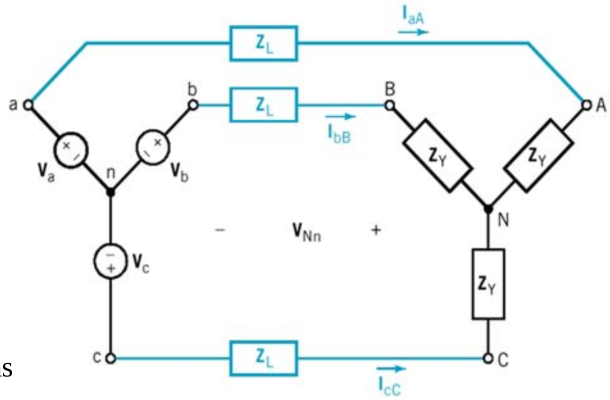


(a)

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 25 + j75 \quad \Omega$$



(c)



(b)

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_L + \mathbf{Z}_Y} = 1.26\angle -66^\circ \ \mathrm{A_{rms}}$$

Example 12.7-1 (cont.)

$$\mathbf{I}_{bB} = 1.26\angle -186^\circ \ \mathrm{A_{rms}} \quad \text{and} \quad \mathbf{I}_{cC} = 1.26\angle -54^\circ \ \mathrm{A_{rms}}$$

The voltages in the per-phase equivalent circuit are

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_Y = 99.6\angle 5^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_{BN} = 99.6\angle -115^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_{CN} = 99.6\angle 125^\circ \quad \mathrm{V_{rms}}$$

The line-to-line voltages are

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 172\angle 35^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 172\angle -85^\circ \quad \mathrm{V_{rms}}$$
$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = 172\angle 155^\circ \quad \mathrm{V_{rms}}$$

$\Rightarrow$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = 0.727\angle -36^\circ \quad \mathrm{A_{rms}}$$
$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = 0.727\angle -156^\circ \quad \mathrm{A_{rms}}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = 0.727\angle 84^\circ \quad \mathrm{A_{rms}}$$

Instantaneous and Average Power in BTP Circuits

One advantage of three-phase power is the smooth flow of energy to the load.

The instantaneous power

$v_{ab} = V\cos\omega t,\; v_{bc} = V\cos(\omega t - 120°),$
and $v_{ca} = V\cos(\omega t - 240°)$

$$\cos^2\omega t = \frac{(1+\cos 2\omega t)}{2}$$

$$p(t) = \frac{v_{ab}^2}{R} + \frac{v_{bc}^2}{R} + \frac{v_{ca}^2}{R}$$

$$= \frac{V^2}{2R}\left[1 + \cos 2\omega t + 1 + \cos 2(\omega t - 120°) + 1 + \cos 2(\omega t - 240°)\right]$$

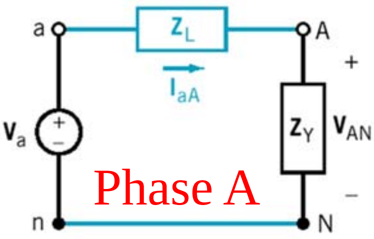
$$= \frac{3V^2}{2R} + \frac{V^2}{2R}\underbrace{\left[\cos 2\omega t + \cos(2\omega t - 240°)\cos(2\omega t - 480°)\right]}_{0}$$

$$= \frac{3V^2}{2R}$$

Instantaneous and Average Power in BTP Circuits

The total average power delivered to the balanced Y-connected load is

$$\mathbf{I}_{aA} = I_L\angle\theta_{AI},\; \mathbf{V}_{AN} = V_P\angle\theta_{AV}$$

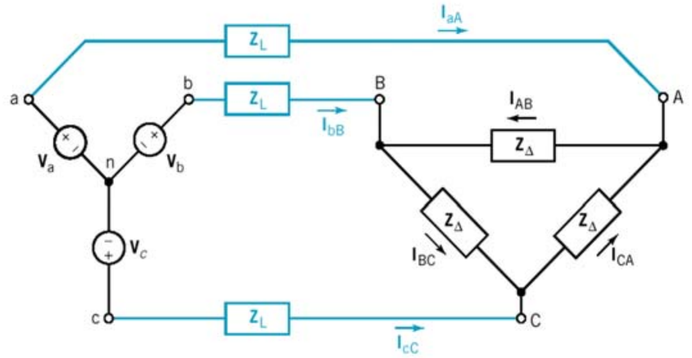


(c)

$$P_Y = 3P_A = 3V_P I_L\cos(\theta_{AV} - \theta_{AI})$$
$$= 3V_P I_L\cos(\theta)$$
$$= 3\frac{V_L}{\sqrt{3}}I_L\cos(\theta)\Big)$$
$$= \sqrt{3}V_L I_L\cos(\theta)$$

Instantaneous and Average Power in BTP Circuits

The total average power delivered to the balanced Δ-connected load is



(a)

$$P_Δ = 3P_{AB} = 3V_{AB}I_{AB}\cos(\theta)$$
$$= 3(\sqrt{3}V_P)\frac{I_L}{\sqrt{3}}\cos(\theta\Big)$$
$$= 3V_P I_L \cos(\theta)$$

Example 12.8-1 *P* = ?

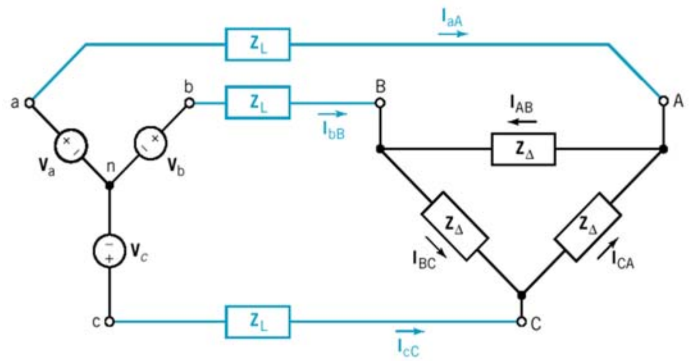
$\mathbf{V}_a = 110\angle 0° \quad \mathrm{V_{rms}}$

$\mathbf{V}_b = 110\angle -120° \quad \mathrm{V_{rms}}$

$\mathbf{V}_c = 110\angle 120° \quad \mathrm{V_{rms}}$

$\mathbf{Z}_L = 10 + j5 \quad \Omega$

$\mathbf{Z}_Δ = 75 + j225 \quad \Omega$



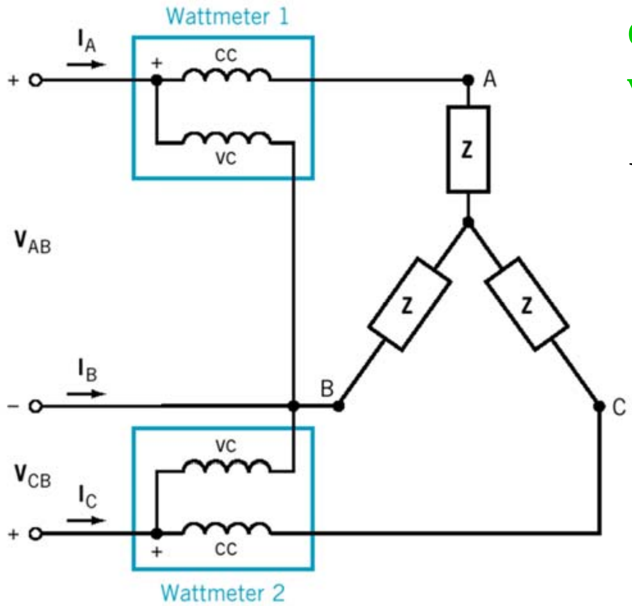
(a)

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_L + \mathbf{Z}_Y} = 1.26\angle -66° \ \mathrm{A_{rms}}$$

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_Y = 99.6\angle 5° \quad \mathrm{V_{rms}}$$

$$P = 3(99.6)(1.26)\cos(5° - (-66°)) = 122.6 \quad \mathrm{W}$$

Two-Wattmeter Power Measurement



cc = current coil
vc = voltage coil

W1 read

$$P_1 = V_{AB} I_A \cos\theta_1$$

W2 read

$$P_2 = V_{CB} I_C \cos\theta_2$$

For balanced load with *abc* phase sequence

$$\theta_1 = \theta_a + 30^\circ \quad \text{and} \quad \theta_2 = \theta_a - 30^\circ$$

θ_a is the angle between phase current and phase voltage of phase *a*

Two-Wattmeter Power Measurement(cont.)

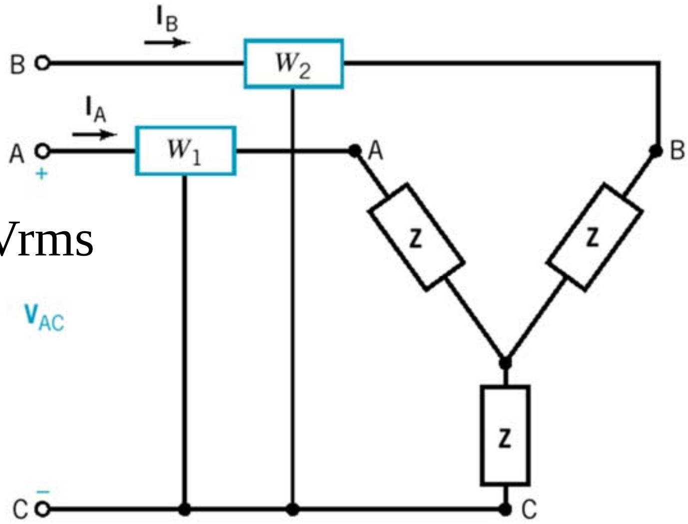
$$P = P_1 + P_2$$
$$= 2V_L I_L \cos\theta \cos 30^\circ$$
$$= \sqrt{3} V_L I_L \cos\theta$$

To determine the power factor angle

$$P_1 + P_2 = V_L I_L 2\cos\theta \cos 30^\circ$$
$$P_1 - P_2 = V_L I_L (-2\sin\theta \sin 30^\circ)$$
$$\frac{P_1 + P_2}{P_1 - P_2} = \frac{V_L I_L 2\cos\theta\cos 30^\circ}{V_L I_L(-2\sin\theta\sin 30^\circ)} = \frac{-\sqrt{3}}{\tan\theta}$$

$$\therefore \tan\theta = \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2} \quad \text{or} \quad \theta = \tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)$$

Example 12.9-1 $P = ?$



$$\mathbf{Z} = 10\angle 45°$$

line-to-line voltage = 220Vrms

The phase voltage

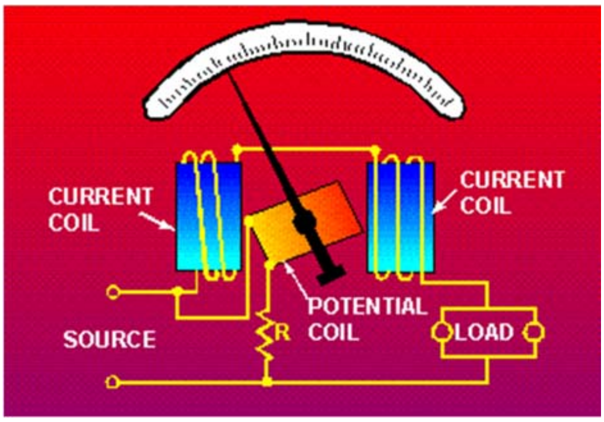
$$\mathbf{V}_A = \frac{220}{\sqrt{3}}\angle -30°$$

The line current

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{\mathbf{Z}} = \frac{220\angle -30°}{10\sqrt{3}\angle 45°} = 12.7\angle -75° \text{ and } \mathbf{I}_B = 12.7\angle -195°$$

$$P_1 = V_{AC} I_A \cos\theta_1 = 2698 \quad \text{W}$$
$$P_2 = V_{BC} I_B \cos\theta_2 = 723 \quad \text{W}$$
$$\Rightarrow P = P_1 + P_2 = 3421 \quad \text{W}$$

Electrodynamic Wattmeter



Digital Power Meter

VAR Meter

***pf* Meter**

Summary

- Three-Phase voltages
- The Y-to-Y Circuits
- The Δ-Connected Source and Load
- The Y-to- Δ Circuits
- Balanced Three-Phase Circuits
- Instantaneous and Average Power in Bal. 3–ϕ Load
- Two-Wattmeter Power Measurement