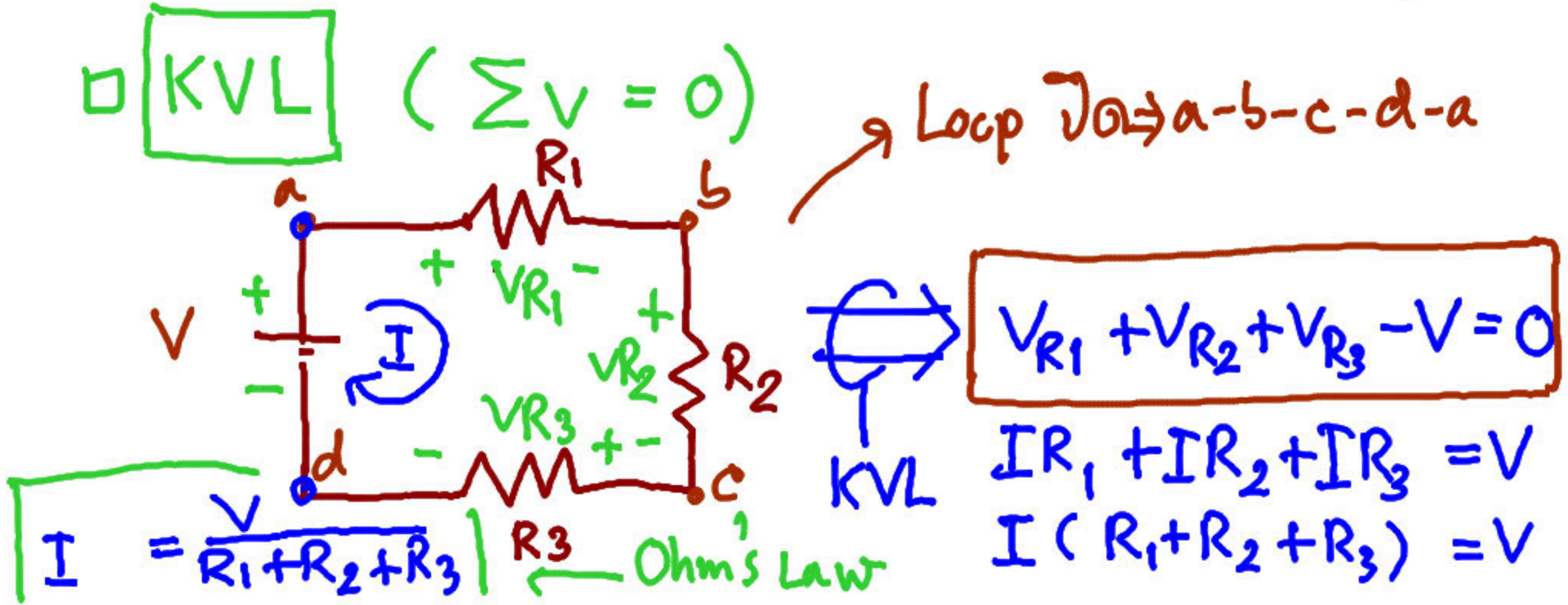
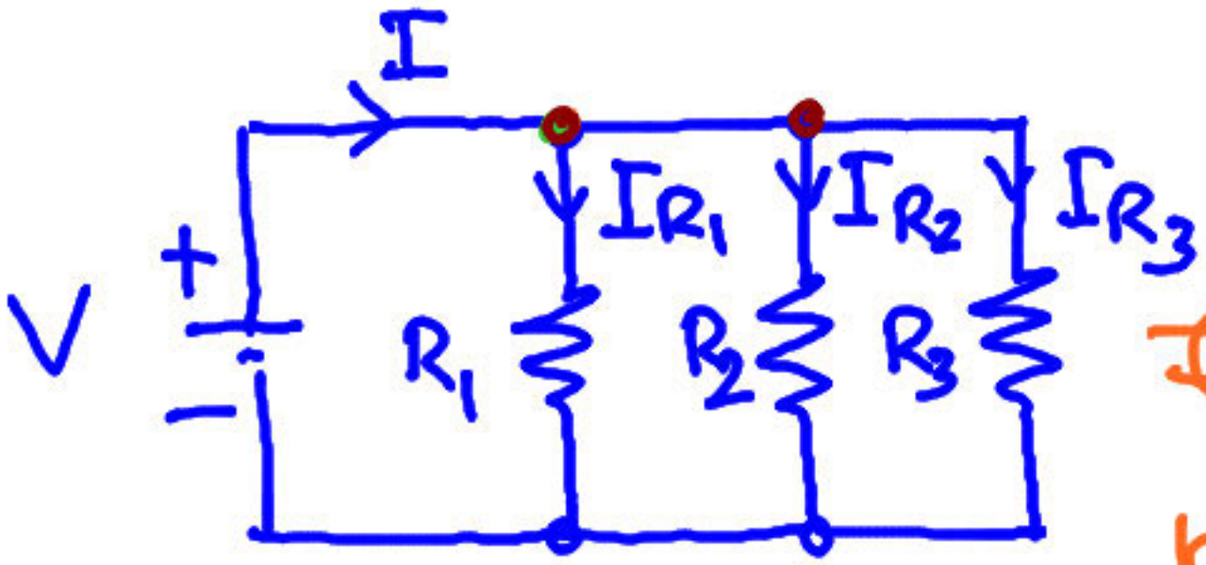
Kirchoff's Law

- Kirchoff's Voltage Law (KVL)
- Kirchoff's Current Law (KCL)



□ **KCL** $(\Sigma I = 0)$ กระแสในลบ = กระแสในบวก



KCL

$$I_{R_1} + I_{R_2} + I_{R_3} - I = 0$$

$$\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = I$$

$$V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = I$$

$$\boxed{V\left(\frac{1}{R_T}\right) = I}$$

Ohm's Law

Cramer's Rule (120) (-20) การแก้สมการที่มีเอ 2 ตัวแปรขึ้นไป

Ex 1 $6x + 4y = 20$ —① หาค่า x, y

$10x + 12y = 30$ —②

สัมประสิทธิ์ ตัวแปร ↓

$$\begin{bmatrix} 6 & 4 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ 10 & 12 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 6 & 4 \\ 10 & 12 \end{bmatrix}^{-1}$$

$$D = \begin{bmatrix} 6 & 4 \\ 10 & 12 \end{bmatrix} \quad -40 \quad 72 \quad = 72 - 40 = 32$$

$$x = \frac{\begin{bmatrix} 20 & 4 \\ 30 & 12 \end{bmatrix}}{D} \quad x \quad = \frac{(20)(12) - (30)(4)}{32} = 120$$

$$y = \begin{bmatrix} 6 & 20 \\ 10 & 30 \end{bmatrix} / D = \frac{(6)(30) - (10)(20)}{32} = -20$$

Ex2 จงหา I_1, I_2 และ I_3

$$\left.\begin{aligned} 10I_1 - 2I_2 - 4I_3 &= 10 \\ -2I_1 + 12I_2 - 6I_3 &= -34 \\ -4I_1 - 6I_2 + 14I_3 &= 40 \end{aligned}\right\}$$

ใช้ S.U.S

$$\underset{A}{\begin{bmatrix} 10 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 14 \end{bmatrix}} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \underset{B}{\begin{bmatrix} 10 \\ -34 \\ 40 \end{bmatrix}}$$

$$[I] = [\ ]^{-1} \times [\ ] =$$

$$D = \begin{vmatrix} 10 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 10 \end{vmatrix}$$

192, 360, 56

1680, −48, −48

$= 1680 - 48 - 48 - (192 + 360 + 56)$

$= 976$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 10 & -2 & -4 \\ -34 & 12 & -6 \\ 40 & -1 & 10 \end{vmatrix}}{976}$$

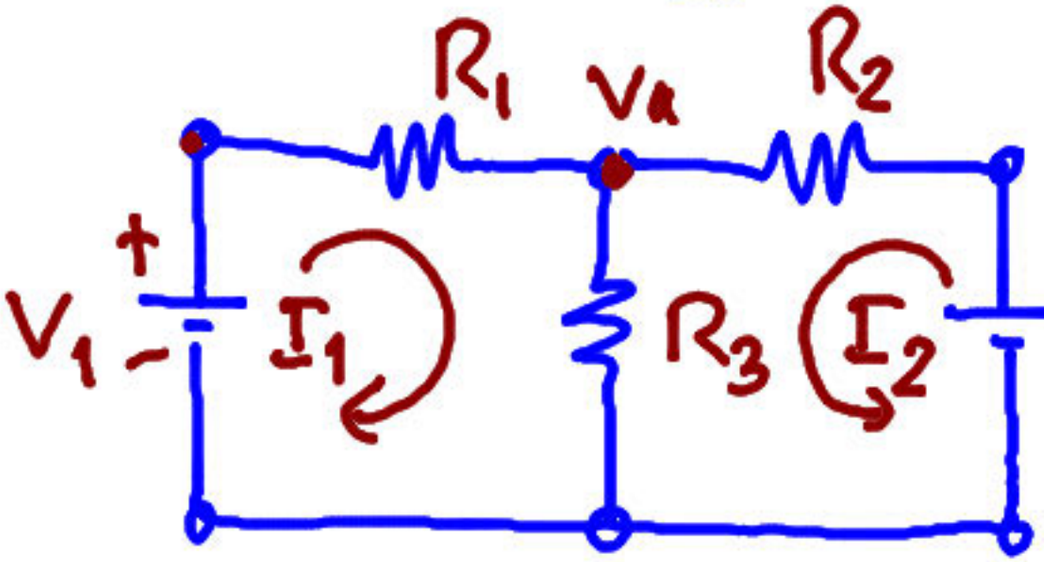
$$= \frac{1952}{976} = 2\ A$$

$I_2 = -1\ A$, $I_3 = 3\ A$

Loop Analysis

(KVL) $\Rightarrow \sum V = 0$

①



Loop #1

$I_1R_1 + I_1R_3 + I_2R_3 = V_1$

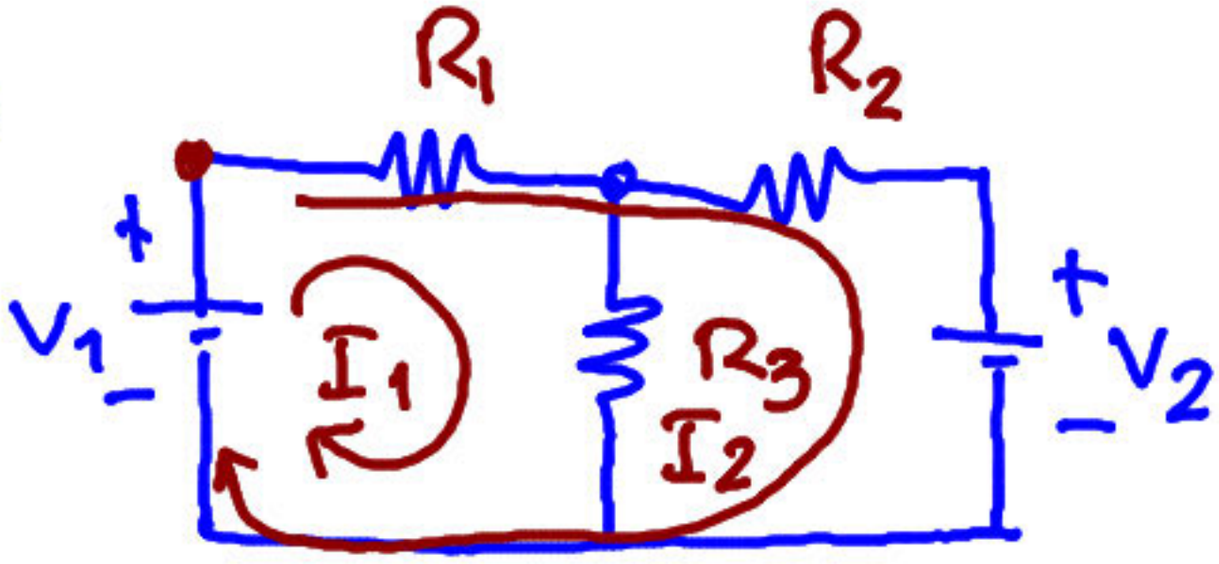
$I_1(R_1+R_3) + I_2R_3 = V_1$

Loop #2

$I_1R_3 + I_2R_2 + I_2R_3 = V_2$

$I_1R_3 + I_2(R_2+R_3) = V_2$

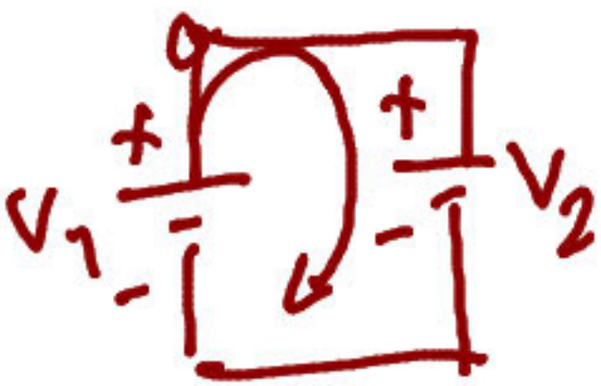
(2)



Loop #1

$$I_1R_1 + I_1R_3 + I_2R_1 = V_1$$

$$I_1(R_1+R_3) + I_2R_1 = V_1$$



$= V_1 - V_2$

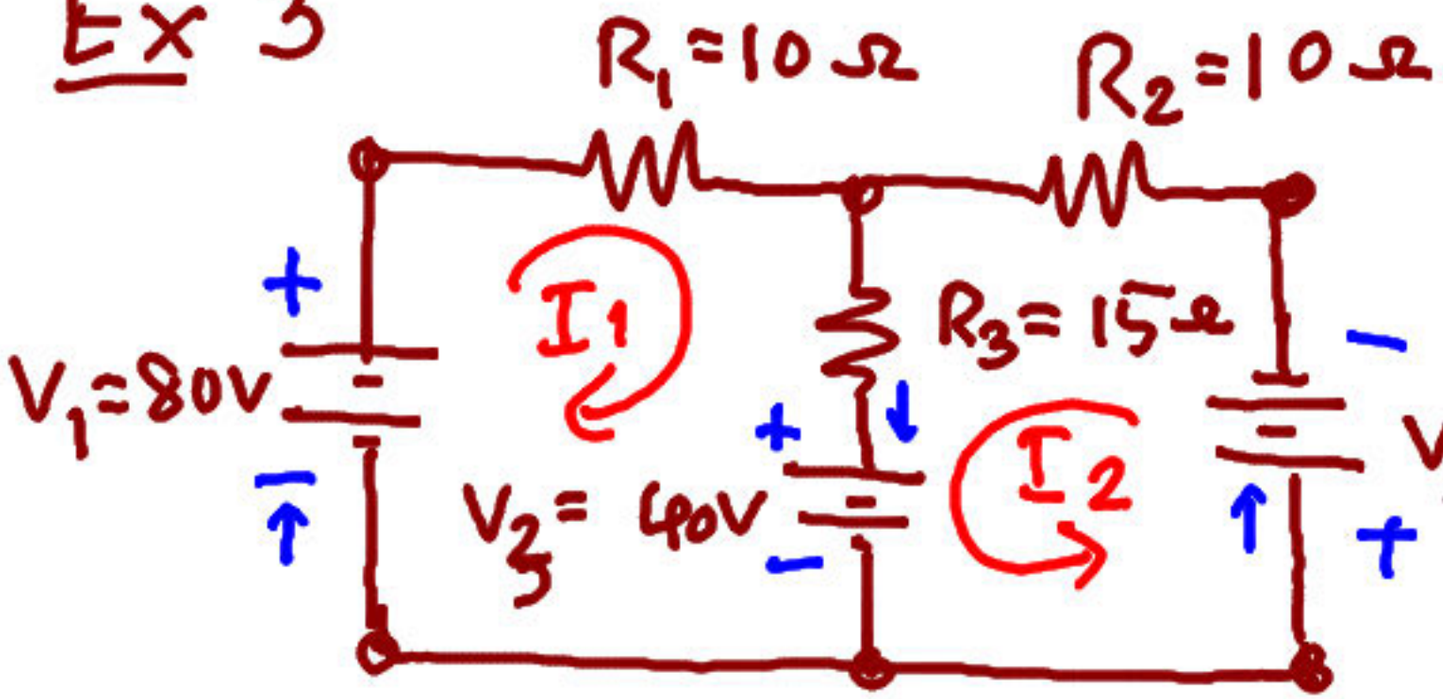
$= V_1 + V_2$

Loop #2

$$I_1R_1 + I_2R_1 + I_2R_2 = V_1 - V_2$$

$$I_1R_1 + I_2(R_1+R_2) = V_1 - V_2$$

Ex 3



จงหากระแสที่ไหลผ่านตัวต้านทานทุกตัว

→ อาศัยกฎของกระแสเคอร์ชอฟ

→ Loop #1 $\Rightarrow$ $I_1R_1 + I_1R_3 + I_2R_3 - V_1 + V_3 = 0$

$$I_1(R_1+R_3) + I_2R_3 = V_1 - V_3$$

$$I_1(25) + I(15) = 80-40$$

$$25I_1 + 15I_2 = 40$$

$$5I_1 + 3I_2 = 8 \quad -(1)$$

Loop #2

$I_1R_3 + I_2R_2 + I_2R_3 + V_2 + V_3 = 0$

$I_1R_3 + I_2(R_2+R_3) = -V_2 - V_3$

$$15I_1 + 25I_2 = -200$$

จากสมการ

$$3I_1 + 5I_2 = -40 \quad \text{---②}$$

$$5I_1 + 3I_2 = 8 \quad \text{---①}$$

แก้สมการ 2 ตัวแปร หา I_1, I_2

$$\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8 \\ -40 \end{bmatrix}$$

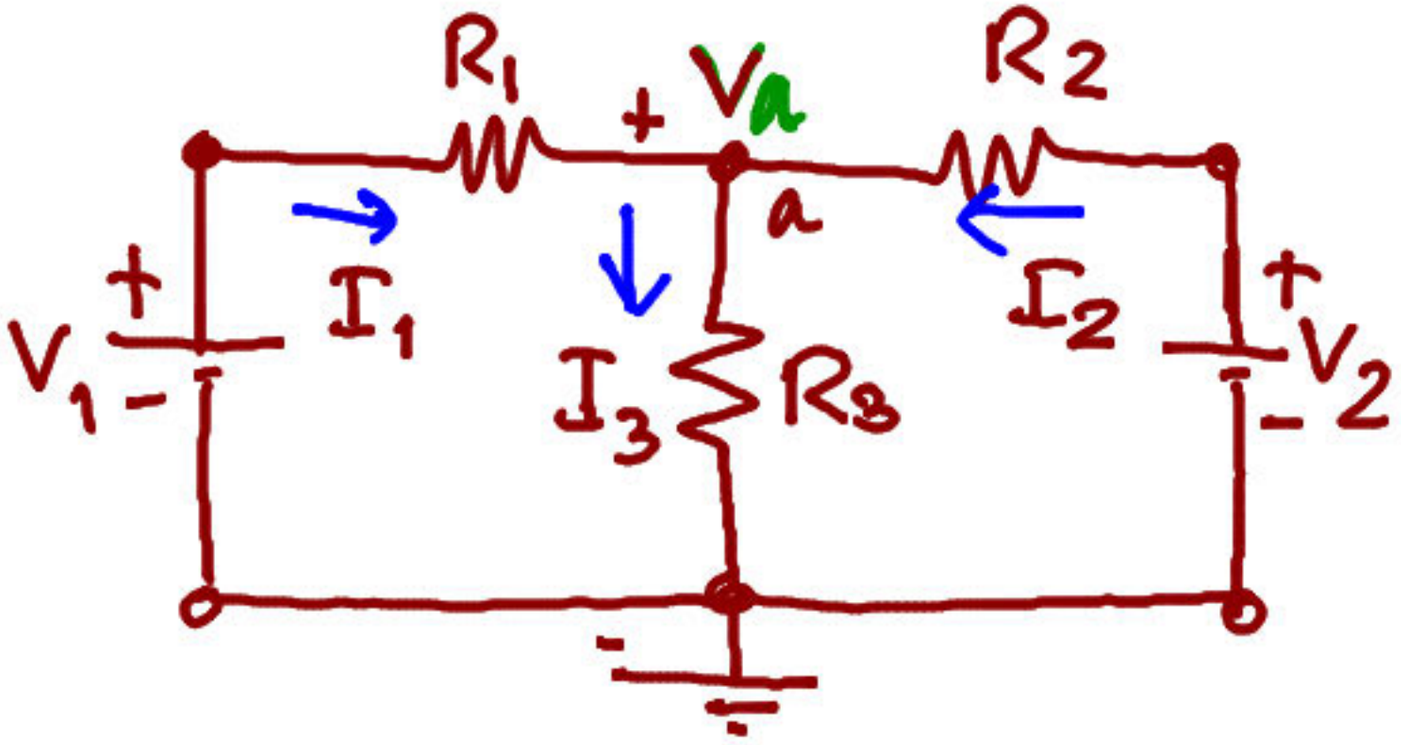
$$I_1 = \frac{\begin{bmatrix} 8 & 3 \\ -40 & 5 \end{bmatrix}}{16} = \frac{40 - (-120)}{16} = \frac{160}{16} = 10\ A$$

$$I_2 = \frac{\begin{bmatrix} 5 & 8 \\ 3 & -40 \end{bmatrix}}{16}$$

$$= \frac{-200 - 24}{16} = -14A$$

I_2 ติดลบ แสดง ว่า เราสมมติ ทิศทาง I_2 ผิด

□ **Node Analysis** (KCL) $\Rightarrow \Sigma I = 0$



KCL

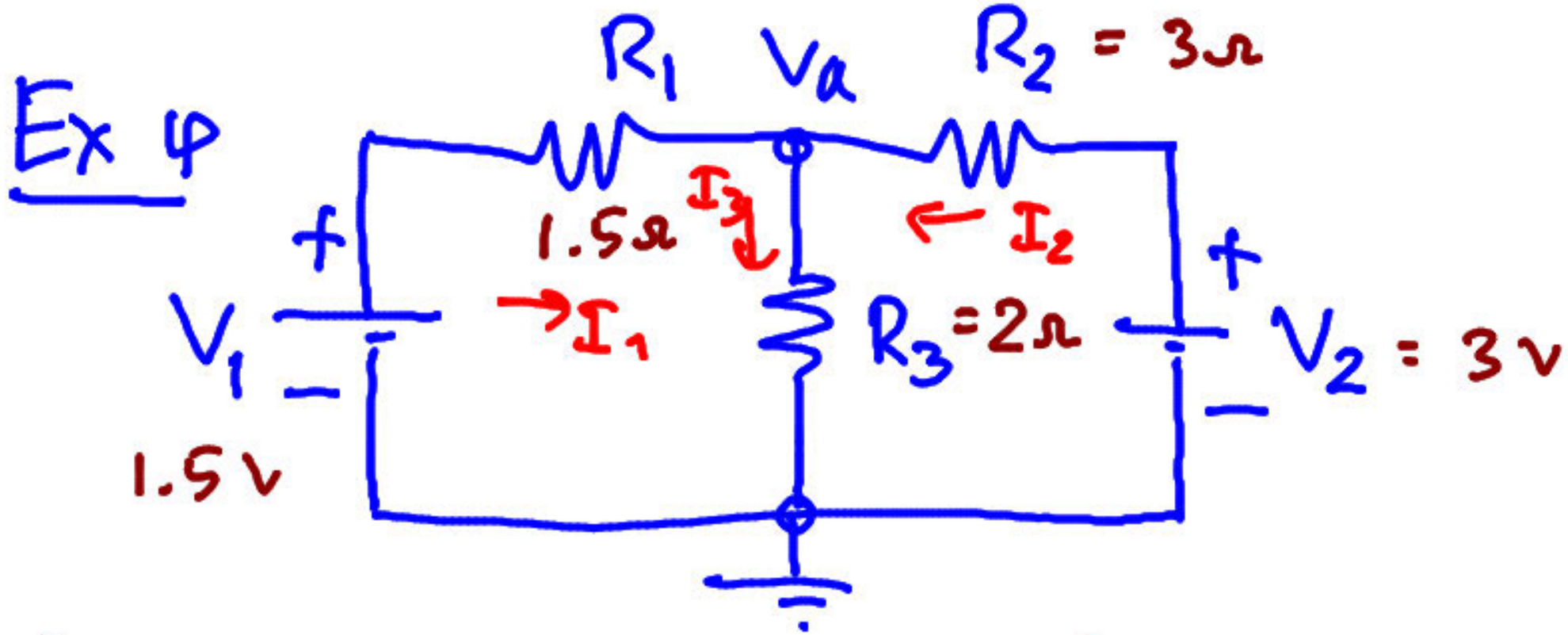
Node V_a ($\Sigma I = 0$)

$I_1 - I_2 - I_3 = 0$

$\frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} - \frac{V_a}{R_3} = 0$

$\frac{V_1}{R_1} - \frac{V_a}{R_1} + \frac{V_2}{R_2} - \frac{V_a}{R_2} - \frac{V_a}{R_3} = 0$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V_a}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3}$$



ให้ $V_1 = 1.5\,V$, $V_2 = 3\,V$, $R_1 = 1.5\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 2\,\Omega$

จงหา I_1 , I_2 และ I_3

วิธีทำ

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V_a}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3}$$

$$\frac{1.5}{1.5} + \frac{3}{3} = \frac{V_a}{1.5} + \frac{V_a}{3} + \frac{V_a}{2}$$

$$2 = \left[\frac{4+2+3}{6}\right] V_a$$

$$2 = \frac{9}{6} V_a$$

$V_{R_1} = V_1 - V_a$

$V_{R_2} = V_2 - V_a$

$V_{R_3} = V_a$

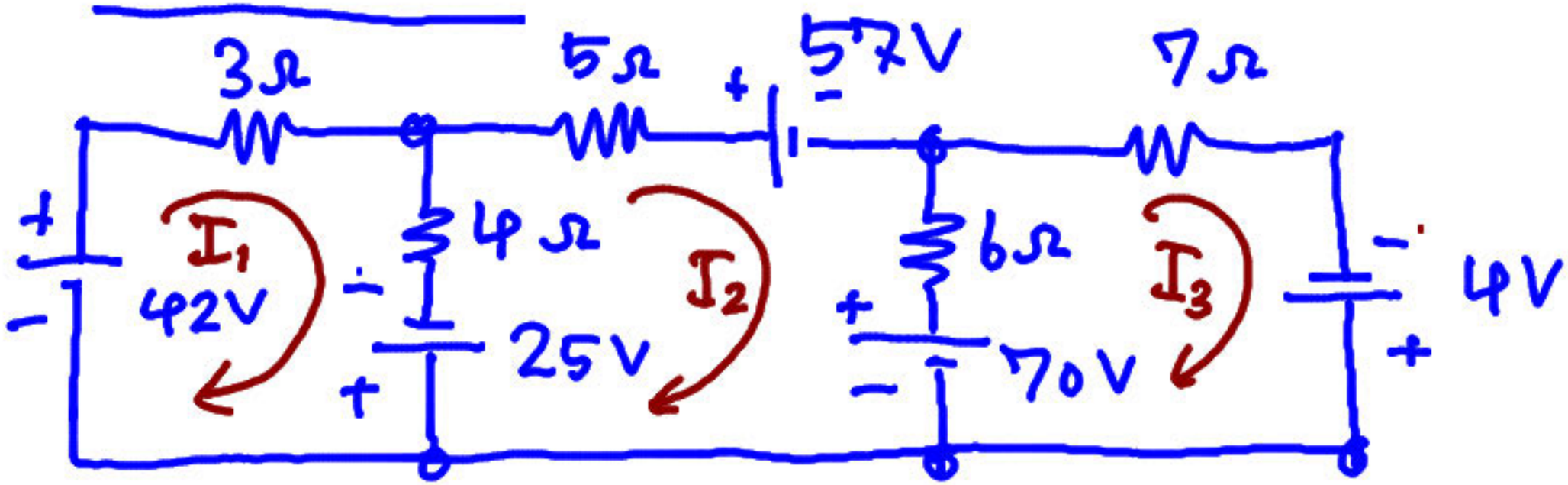
$$V_a = \frac{12}{9} = 1.333 \text{ Volt}$$

$$\therefore I_1 = \frac{V_1 - V_a}{R_1} = \frac{1.5 - 1.333}{1.5} = 0.111 \text{ A}$$

$$I_2 = \frac{V_2 - V_a}{R_2} = \frac{3 - 1.333}{3} = 0.555 \text{ A}$$

$$I_3 = V_a / R_3 = 1.333/2 = 0.666 \text{ A}$$

ทดสอบ #2



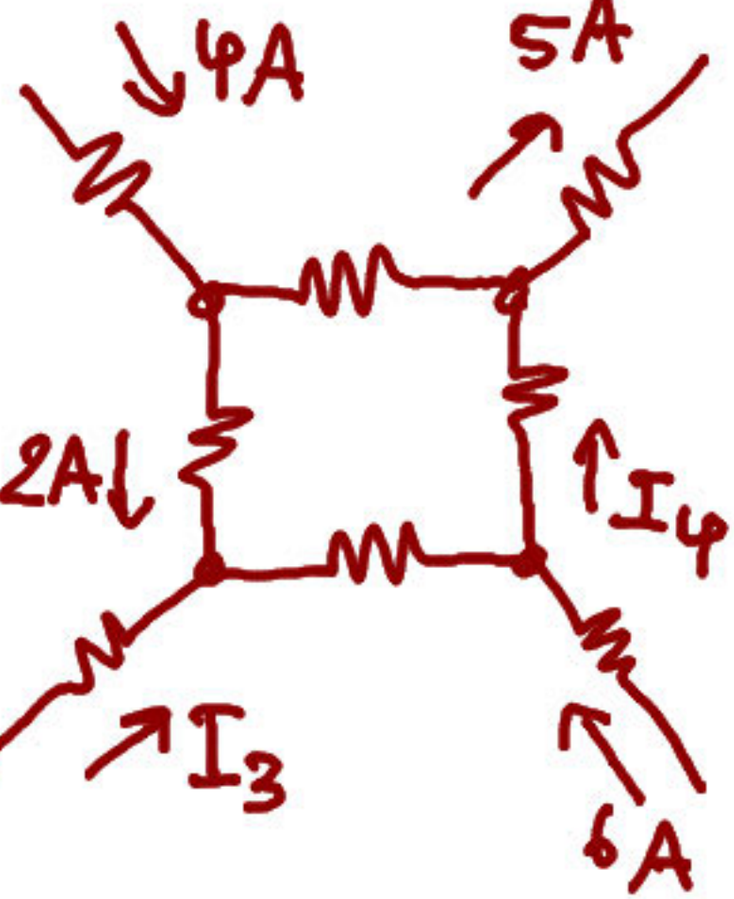
จงหาค่าของ I_1 , I_2 และ I_3

การบ้านครั้งที่ 2

① 8Ω 5Ω 2Ω 30V 20V 100V 9Ω a b V_{ab} I

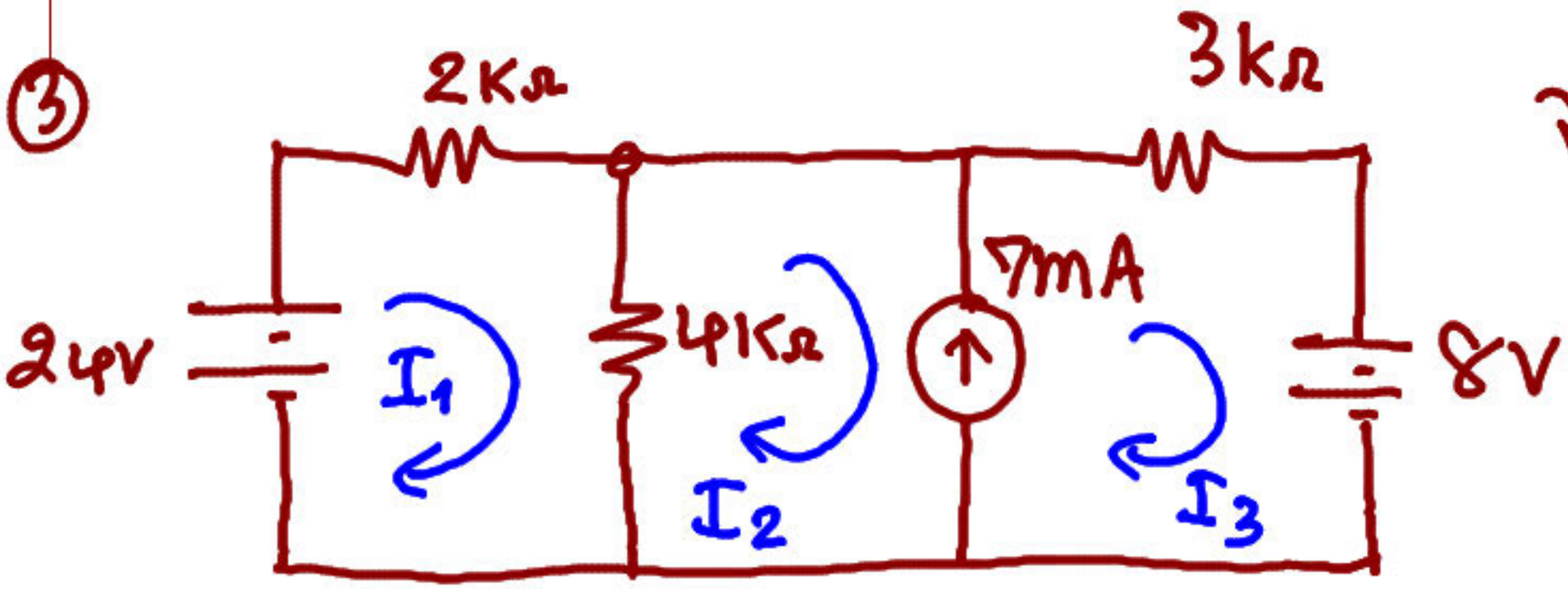
จงหา I และ: V_{ab}

②



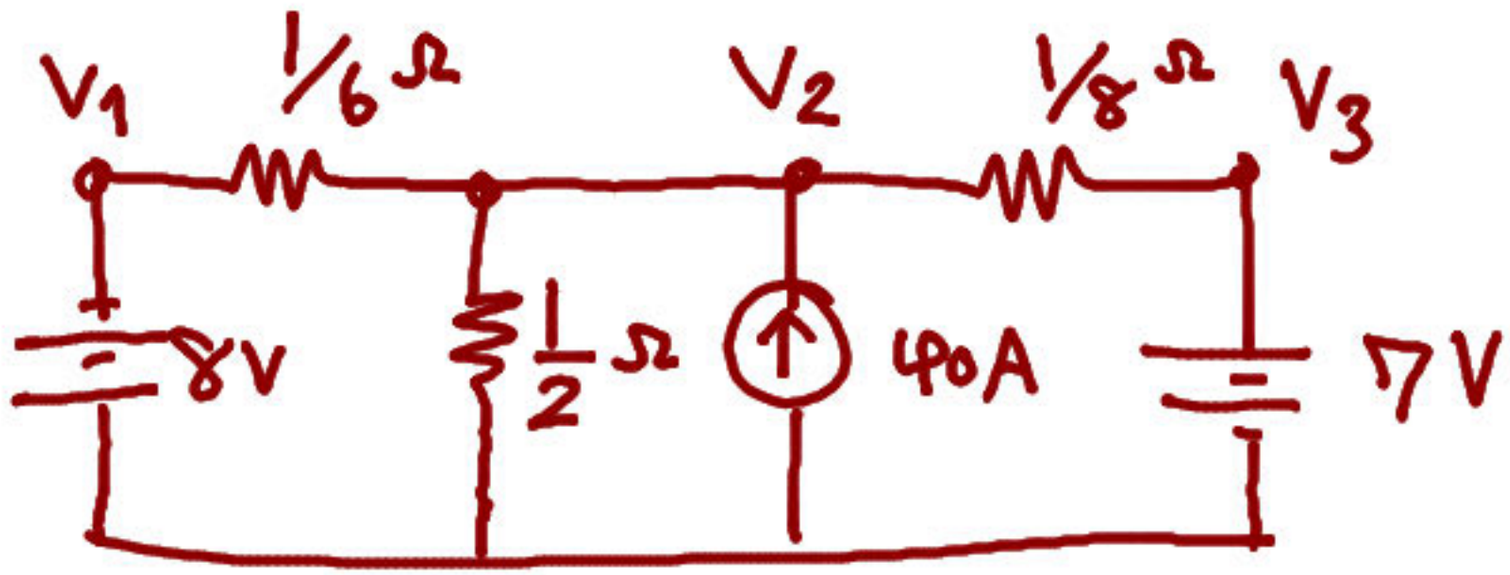
จงหา I_3, I_4

③



จงหา I_1, I_2, I_3

④



จงหา V_1, V_2 และ V_3