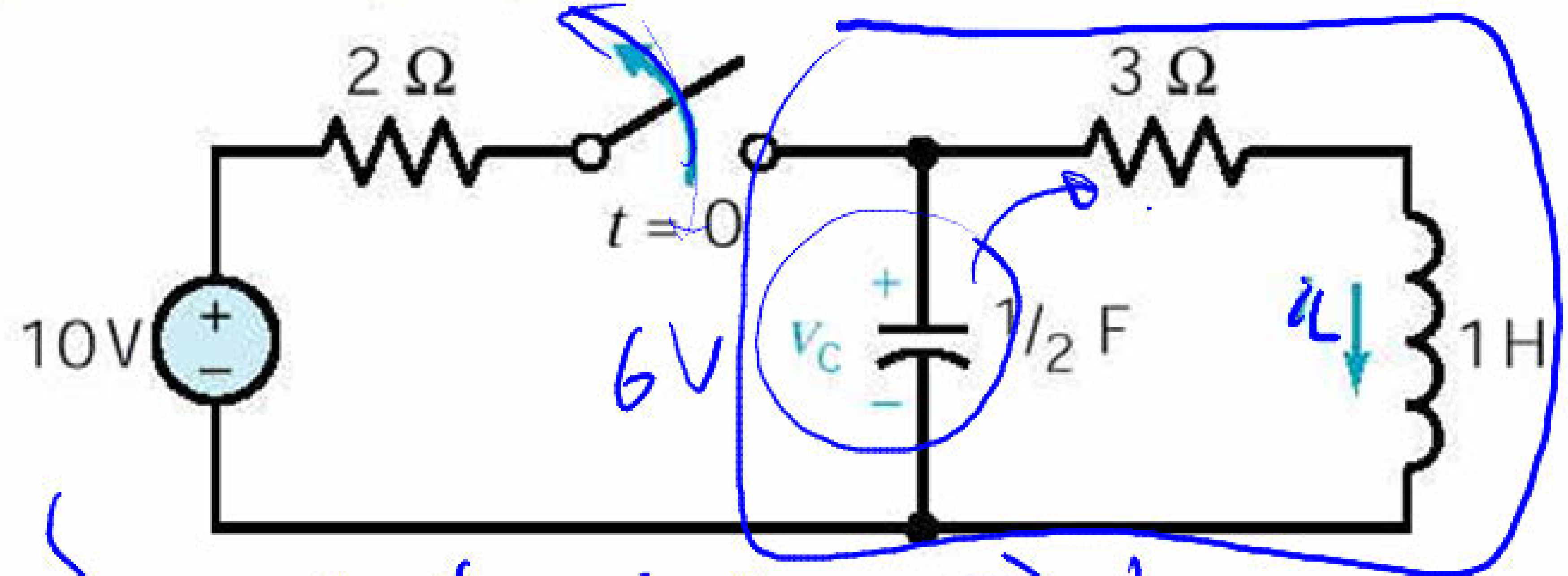
# DC Steady-State Property of Inductors

## Example:

Find the inductor current at $t = 0^-$ and $0^+$. Assume that the switch has been closed for a very long time.

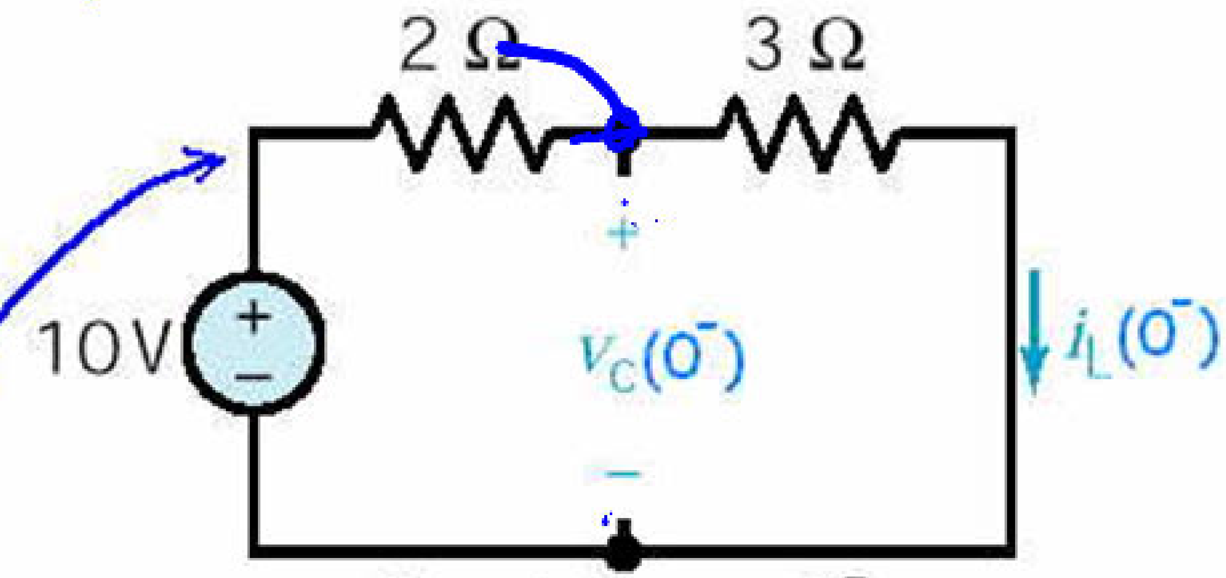



$v(0^+) = V(0^-)$ ⇓ C

$i(0^+) = i(0^-) \Rightarrow L$

## Solution:

Circuit operates under dc steady-state mode for negative time (close to zero), since source is dc and switch has been closed for a long time.




Circuit at $t = 0^-$

Therefore, the capacitor behaves as open circuit and the inductor behaves as a short circuit.

By ohm's law:

$i_L(t = 0^-) = 10/(2+3) = 2\ A$

By inductor current continuity:

$i_L(t = 0^+) = i_L(t = 0^-) = 2A$

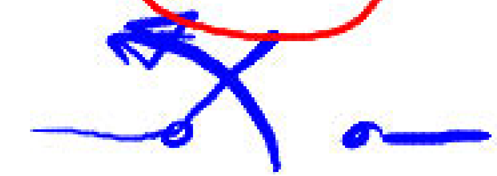
$\boxed{i_L(t = \infty) = ?}$

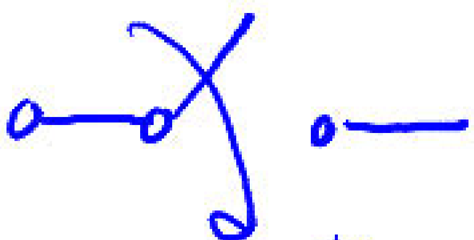
# Computing Initial Conditions: Example

Find $i_L(0^+)$, $v_c(0^+)$, $\dfrac{dv_c(0^+)}{dt}$, and $\dfrac{di_L(0^+)}{dt}$ for the following circuit.

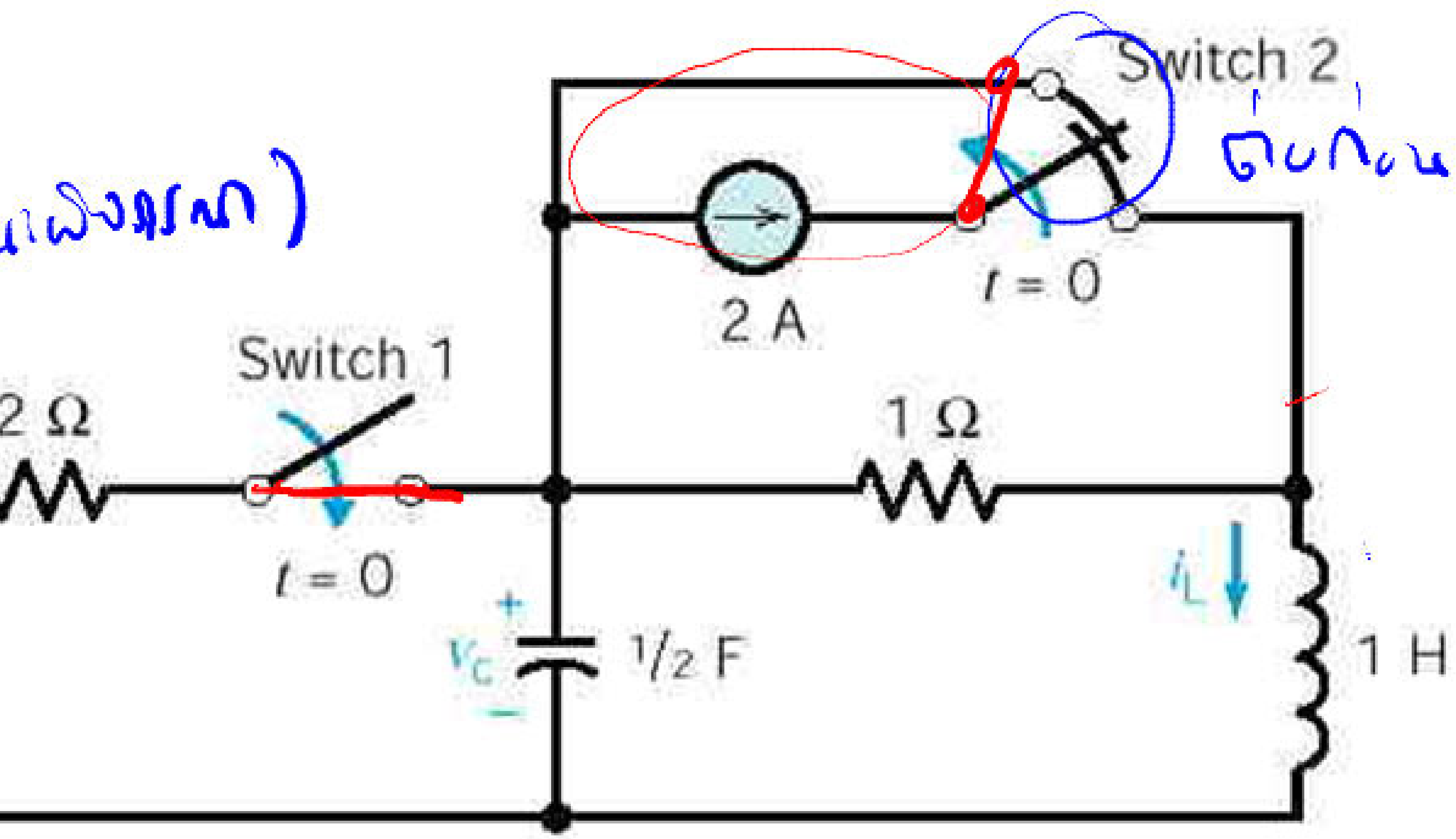
We will use $\dfrac{dv_c(0^+)}{dt}$ to denote $\left.\dfrac{dv_c(t)}{dt}\right|_{t=0^+}$

$t = 0^+$





$t = 0^+$

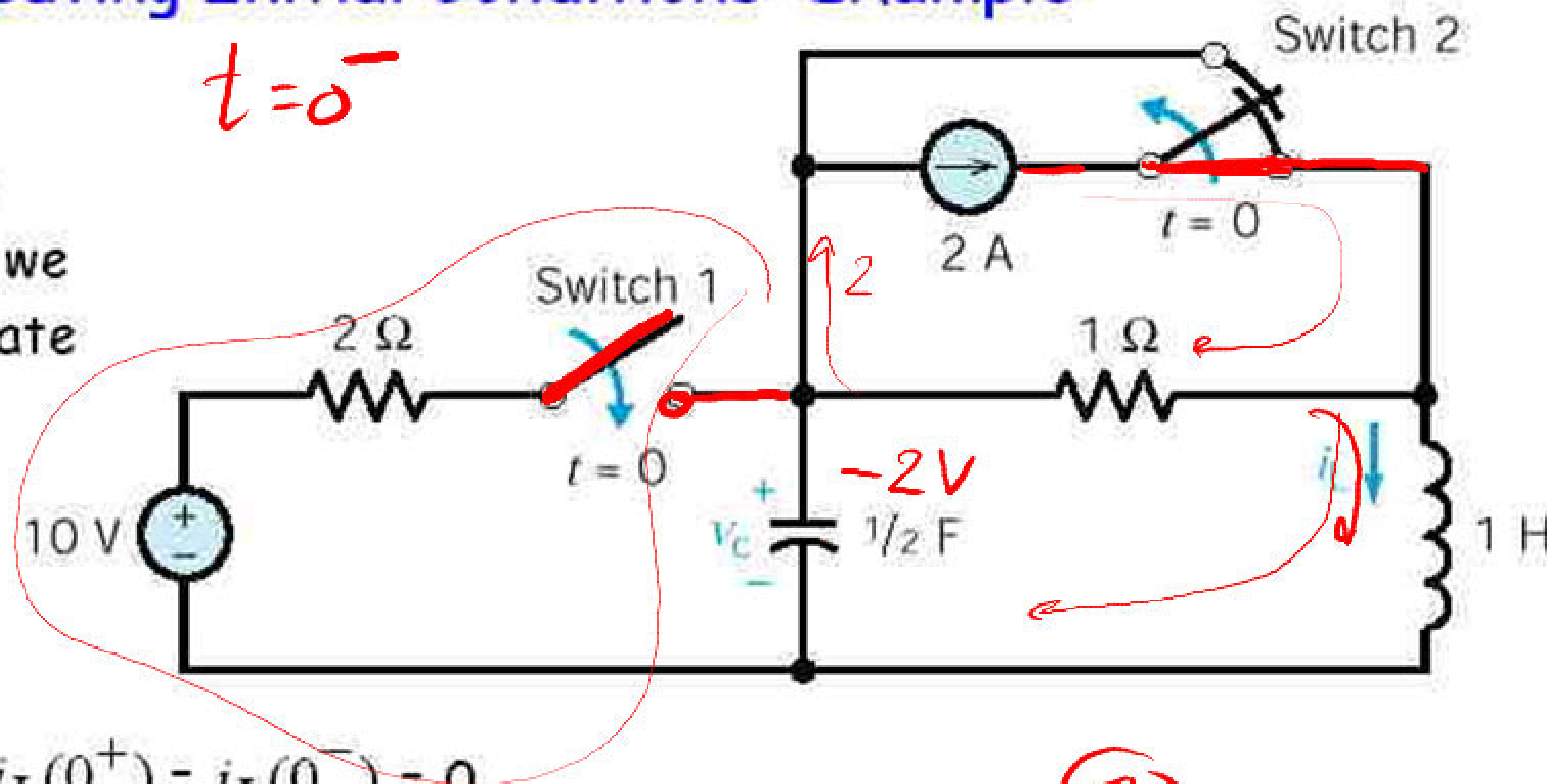


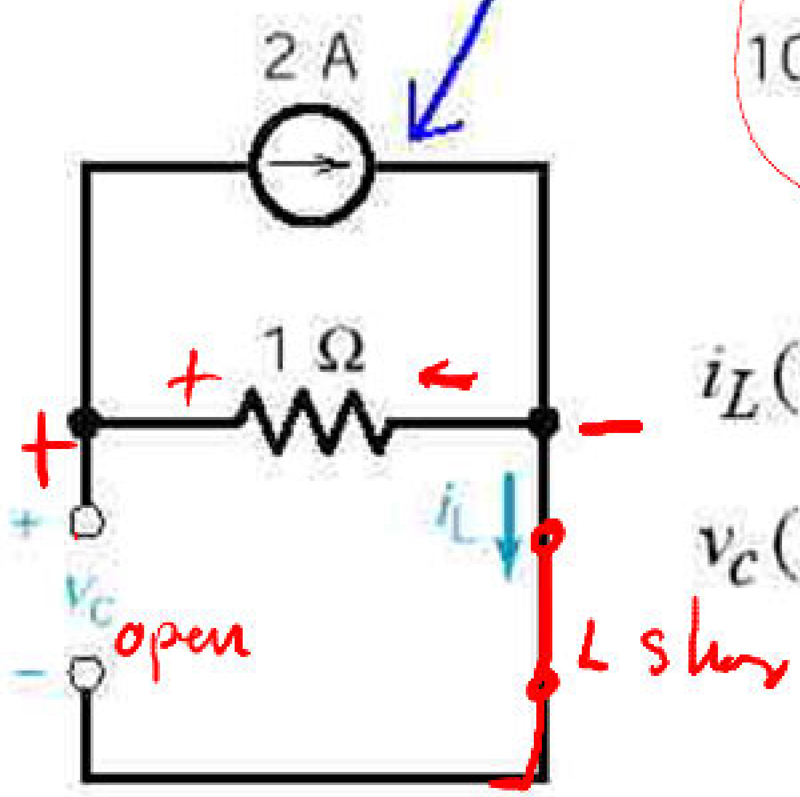
# Computing Initial Conditions: Example

## Solution

$t = 0^-$

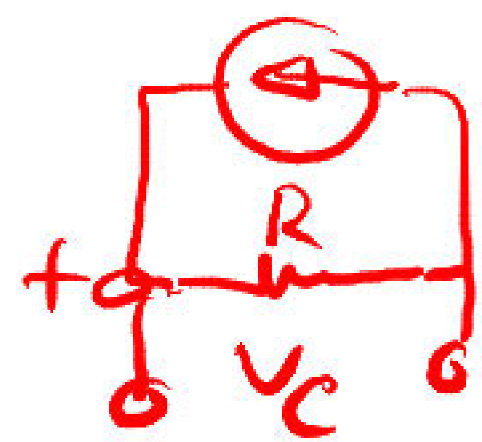
First, we redraw the circuit for $t = 0^-$ and we assume dc steady-state operation.




$$i_L(0^+) = i_L(0^-) = 0$$

$$v_C(0^+) = v_C(0^-) = -(2)(1) = -2 \text{ V.}$$




In order to find the derivatives $\dfrac{dv_C(0^+)}{dt}$, and $\dfrac{di_L(0^+)}{dt}$ we need to deal with the circuit at $t = 0^+$.

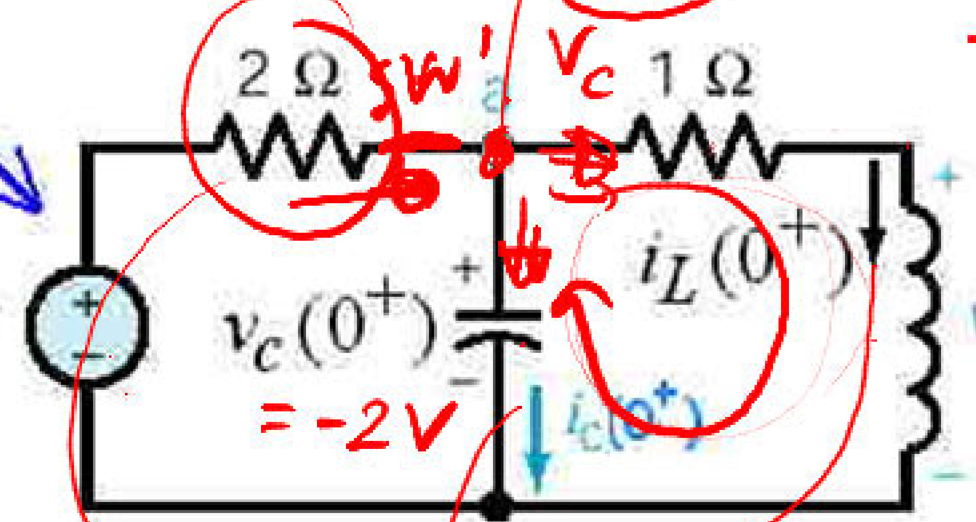
# Computing Initial Conditions: Example

Solution (cont.)    Sw. 1 ปิด , Sw 2 เปิด

In order to find the derivatives $\frac{dv_c(0^+)}{dt}$, and $\frac{di_L(0^+)}{dt}$ we need to deal with the circuit at $t = 0^+$

Recall that:

$$\boxed{i_c = C\frac{dv_c}{dt}} \quad \text{so} \quad \boxed{\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}}$$




$i_L$

$i_c$

Similarly for the inductor:

$$\boxed{v_L = L\frac{di_L}{dt}} \quad \text{so} \quad \boxed{\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}}$$

$i_L(0^+) = 0 \qquad v_c(0^+) = -2$ ~~A~~. V

Using KVL for RHS mesh:

$$v_L(0^+) - v_c(0^+) + (1)i_L(0^+) = 0 \checkmark$$

$$v_L(0^+) = v_c(0^+) - i_L(0^+) = -2\text{V}$$

0

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = -2\ \frac{\text{A}}{\text{s}}$$

Using KCL at node a:

−2    −6

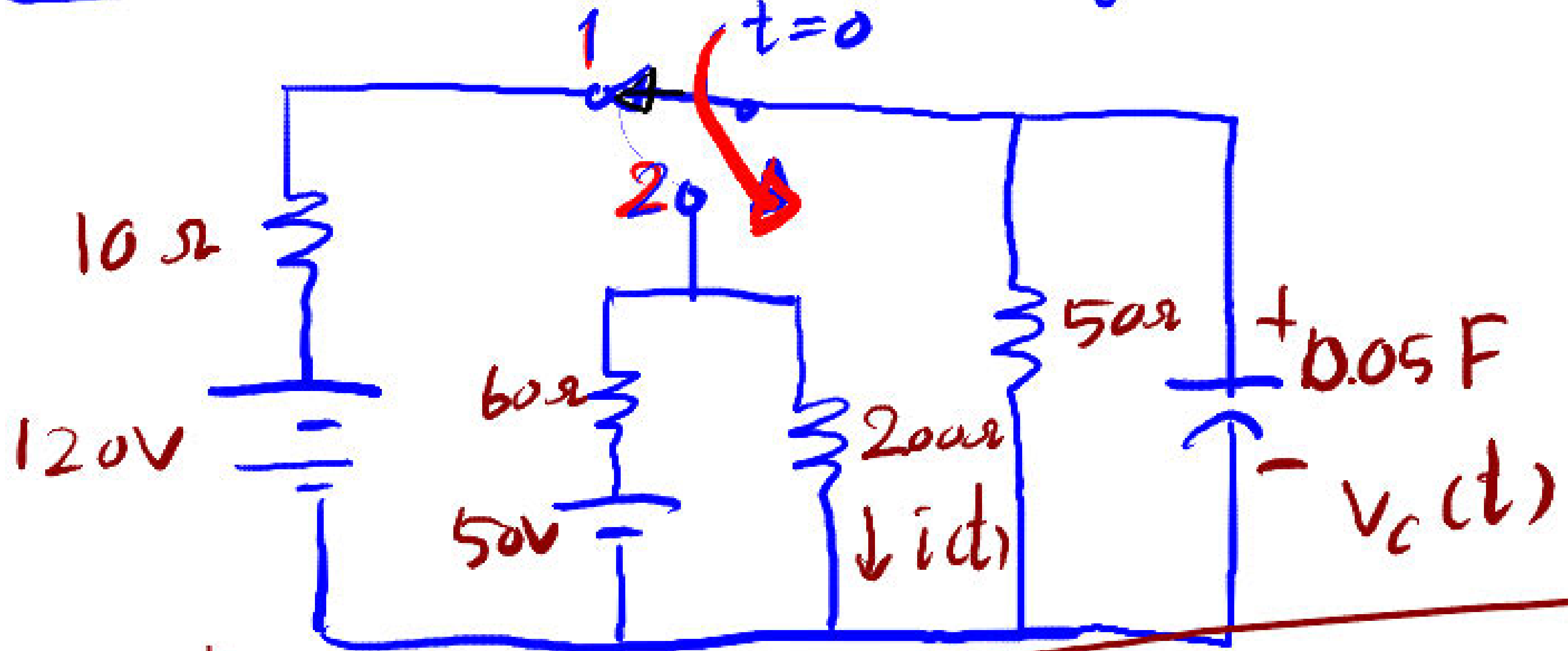
0

$$i_c(0^+) + i_L(0^+) + \frac{v_c(0^+) - 10}{2} = 0$$

$$i_c(0^+) = 6\ \text{A}$$

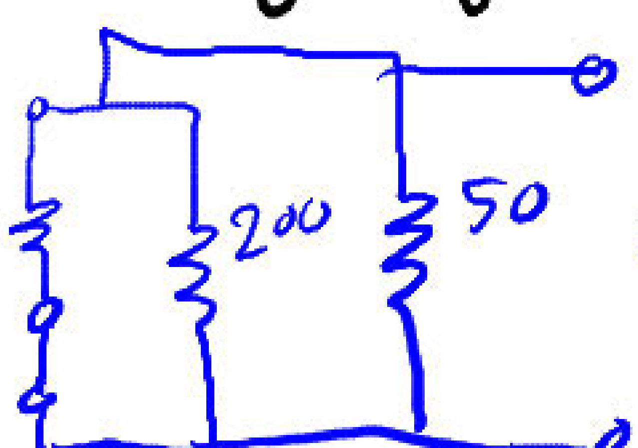
$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{6}{1/2} = 12\ \frac{\text{V}}{\text{s}}$$

Ex 1 จงหาผลตอบสนองสมบูรณ์ $(i(t), v_c(t))$




$$\boxed{f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}, \quad t \geq 0}$$

Step 1 $\tau = R_{eq}C_{eq}$ ; $t \geq 0$ (sw = 2)

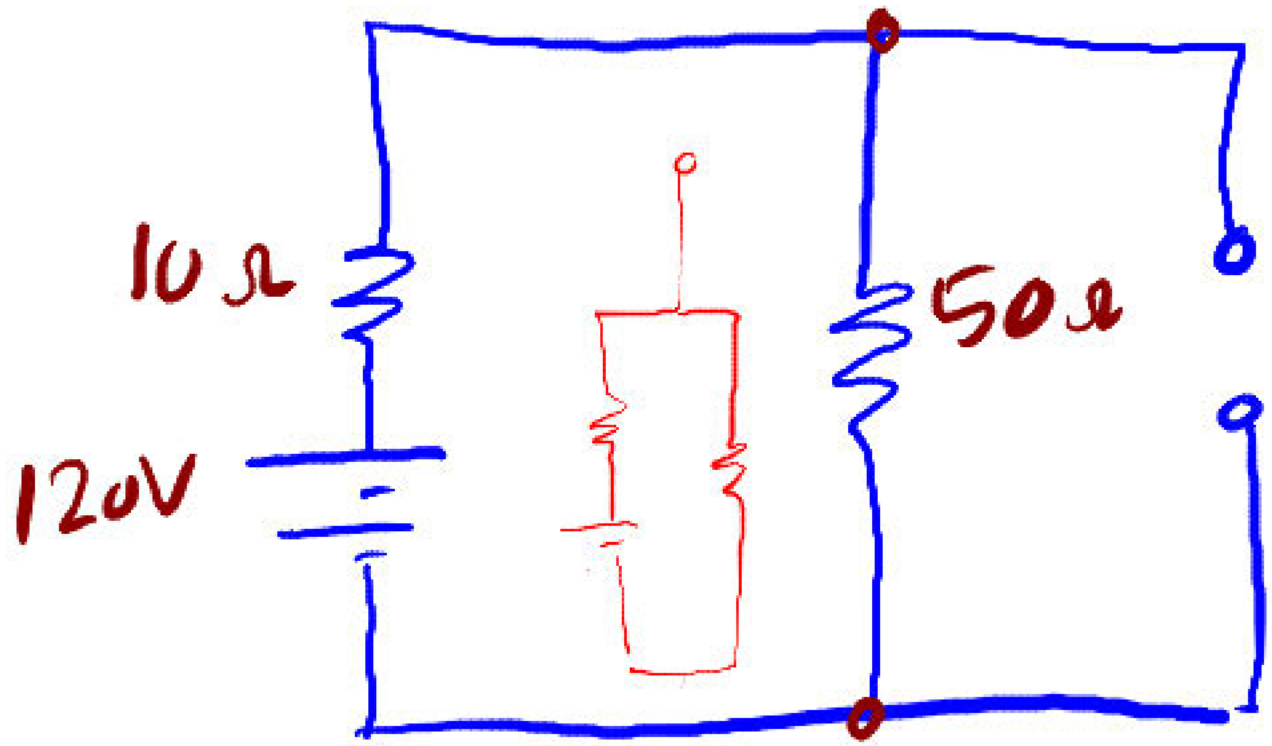



$R_{eq} = R_{eq} = \dfrac{1}{\frac{1}{60} + \frac{1}{200} + \frac{1}{50}} = 24\,\Omega$

$\therefore \tau = 24 \times 0.05$

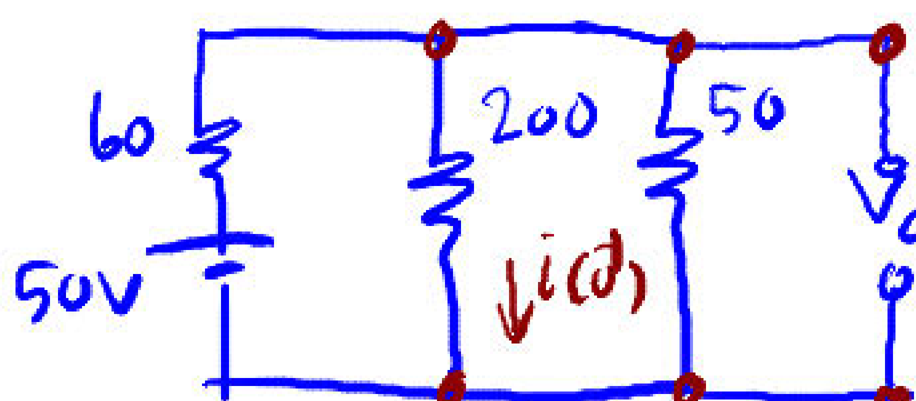
$= 1.2$ Sec.

Step 2 $V_c(0^+) = V_c(0^-)$ من $t<0$;




$$V_c(0^-) = \frac{120 \times 50\Omega}{10\Omega + 50\Omega}$$
$$= 100\ V = V_c(0^+)$$

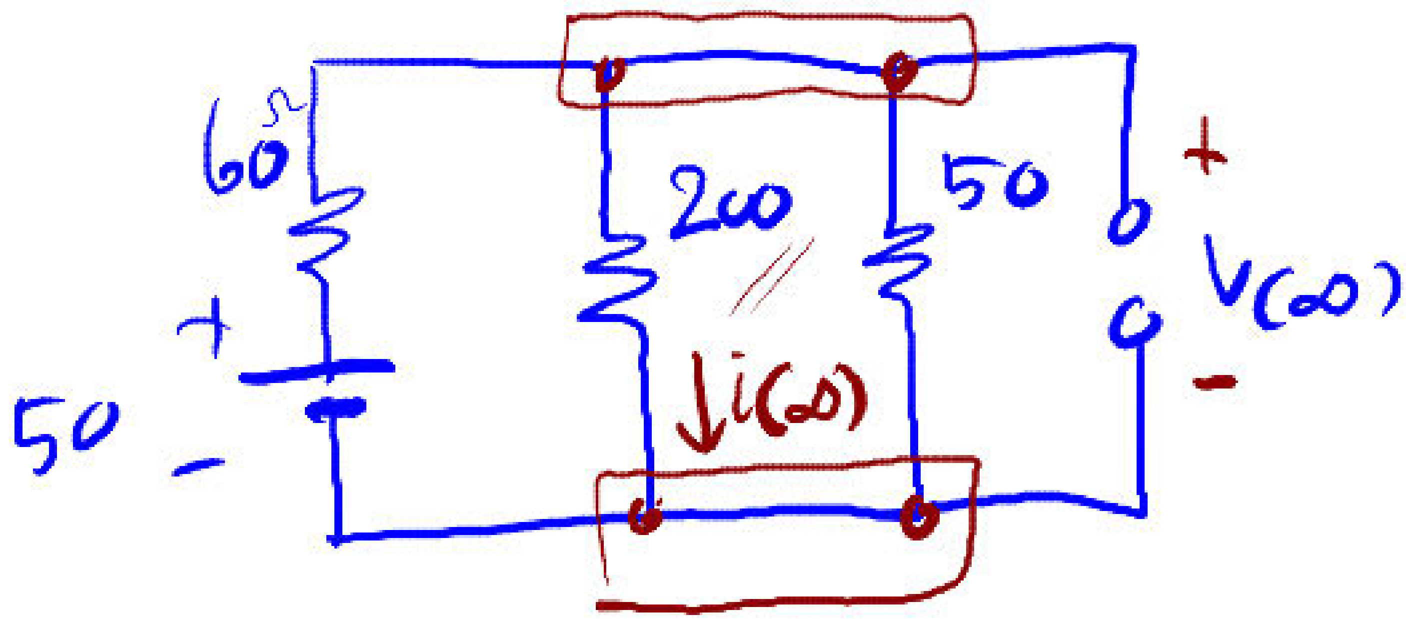
step 3 في $t = 0^+$ من $i(t)$




$V_c(0^+) = 100\ V$

$$\therefore i(0^+) = \frac{100\ V}{200\ \Omega}$$
$$= 0.5\ A$$

step 4 ที่ $t = \infty$ หา $\boxed{v(\infty),\ i(\infty)}$




$$V(\infty) = \frac{50\left(\frac{200\times 50}{200+50}\right)}{\left(\frac{200\times 50}{200+50}\right)+60}$$

$$= 20 \text{ Volt.}$$

$$i(\infty) = \frac{v(\infty)}{R} = \frac{20}{200}$$

$$= 0.1 \text{ A}$$

step 5

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right] e^{-t/\tau}$$

$$= 20 + [100 - 20]\, e^{-t/1.2}\ \text{V};\ t \geq 0$$

$$i(t) = 0.1 + [0.5 - 0.1]\, e^{-t/1.2}\ \text{A};\ t \geq 0$$